Panel 1

Local Extremum Inflection Points, and the like

Basic Theorem: If f has a local extremum at $x=c$, then $f'(c)=0$ or $f'(c)$ d.n.e.

If $f'(x)>0$: f increasing ↗

$f'(x)<0$: f decreasing ↘

$f''(x)>0$: (f' is incr.) ⇒ f concave up ∪

$f''(x)<0$: (f' is decr.) ⇒ f concave down ∩

Def: If $f'(c)=0$ or $f'(c)$ d.n.e. → x is critical

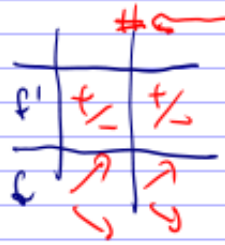
If $f''(c)=0$ or $f''(c)$ d.n.e. → x is potential infl. pts

1

Panel 2

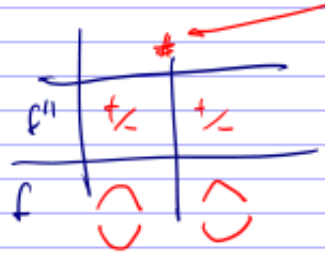
To discuss local extrema, increasing, or decreasing

1. Find f'
2. Find critical pts $f'=0$, f' d.n.e
3. Make "sign" table for f'



To discuss inflection points and concavity

1. Find f''
2. Find possible infl. pts. $f''=0$ or f'' d.n.e
3. Make "sign" table for f'



2

Panel 3

Examples from HW

$f(x) = \frac{y-1}{y^2-y+1}$

$f'(x) = \frac{1(y^2-y+1) - (y-1)(2y-1)}{(y^2-y+1)^2} = \frac{y^2-y+1-2y^2+y+2y-1}{(y^2-y+1)^2}$

$= \frac{-y^2+2y}{(y^2-y+1)^2} = 0$ Critical: $2y-y^2 = y(2-y) = 0$ $y = 0, 2$

or since $y^2-y+1 = 0$

$y = \frac{1 \pm \sqrt{1-4}}{2}$ = not real

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Panel 4

Find all inflection points of $y = x^3 - 3x^2 + 9x - 7$

① $f'(x) = 3x^2 - 6x + 9$

$f''(x) = 6x - 6 = 0$

② Infl. : $x = 1$

③

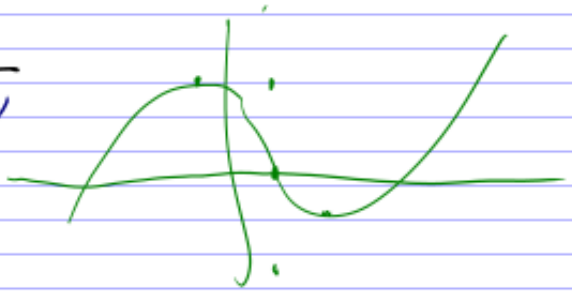
	1	
	0	2
f''	−	+
f	∩	∪

Answer.

infl. point: at $x = 1$, $y = 0$

concave up : $(1, \infty)$

concave down: $(-\infty, 1)$



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Panel 5

Quiz #7 Name: _____

① Find all rel. extrema and intervals where f is increasing/decreasing for $f(x)=3x^4+4x^3-12x^2$

$f'(x) = 12x^3+12x^2-24x$

$= 12x(x^2+x-2)=0$

$= 12x(x-1)(x+2)=0$

$x=0,\ x=1,\ x=-2$

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Panel 6

② Find all inflection points and intervals where f is concave up or down for $f(x)=x^4-6x^2$

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Panel 7

Curve sketching

① domain
② intersects (with y-axis) (with x-axis if possible)
③ asymptotes
④ critical points
⑤ possible infl. points
⑥ THE TABLE
⑦ The values
⑧ The graph

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Panel 8

Ex: Sketch the graph of $f(x)=3x^4+4x^3-12x^2$

① domain: $\mathbb{R}$
② y-intercept: $(0,0)$
③ asympt.: none
④ $f'=12x^3+12x^2-24x=12x(x-1)(x+2)=0$
$x=0,+1,-2$
⑤ $f''=36x^2+24x-24=12(3x^2+2x-2)=0$
$x=\frac{-2\pm\sqrt{4+24}}{6}=\frac{-2\pm\sqrt{28}}{6}=$

① domain
② intersects
③ asymptotes
④ critical points
⑤ possible infl. points
⑥ THE TABLE
⑦ The values
⑧ The graph

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Panel 9