Panel 1

$$f(x) = x^5 \cdot \sin\left((3x+2)^4\right)$$

$$f'(x) = (5x^4)\left(\sin\left((3x+2)^4\right)\right) + (x^5)\left(\quad\right)$$

$$\frac{d}{dx} \cancel{\sin}\left((3x+2)^4\right) = \cos\left((3x+2)^4\right) \cdot 4(3x+2)^3 \cdot 3$$

1

Panel 2

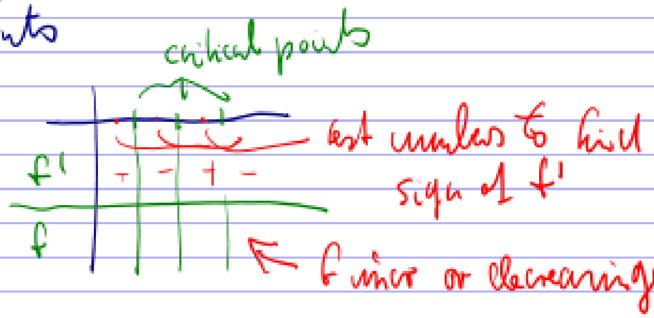
Finding relative extrema

① Find $f'(x)$

② Solve $f'(x) = 0$, and $f'(x)$ does not exist

$\Rightarrow$ critical points

③ Create table



2

Panel 3

Ex: Find local extrema for $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ and identify all intervals where f is increasing

① $f'(x) = 12x^3 - 12x^2 - 24x$

② $f'(x) = 0 = 12x(x^2 - x - 2) = 12x(x-2)(x+1)$

critical: $x = 0, 2, -1$

③

	-2000	$-\frac{1}{2}$	1	2000
f'	$-$	$+$	$-$	$+$
f	$\searrow$	$\nearrow$	$\searrow$	$\nearrow$

$x = -1$, rel. min

$x = 0$, rel. max

$x = 2$: rel min

answer $(-1, 0) \cup (2, \infty)$

3

Panel 4

Ex: Find relative extrema of $f(x) = x^3(1 - x^2)$

$x^3 - x^5$

$f'(x) = 3x^2 - 5x^4 = x^2(3 - 5x^2) = 0$

$x = 0$, $x = \pm\sqrt{\frac{3}{5}}$

$3 - 5x^2 = 0$

$3 = 5x^2$

$\frac{3}{5} = x^2$

$\pm\sqrt{\frac{3}{5}} = x$

	-1000	$-\sqrt{\frac{3}{5}}$, -0.0001	0, 0.0001	$\sqrt{\frac{3}{5}}$, 1000
f'	$-$	$+$	$+$	$-$
f	$\searrow$	$\nearrow$	$\nearrow$	$\searrow$

$x = 0$: neither

$x = -\sqrt{\frac{3}{5}}$: min

$\Rightarrow x = +\sqrt{\frac{3}{5}}$: max

4

Panel 5

Ex: Rel. extrema of $f(x) = x^{1/3}(2-x)$

$$f(x) = 2x^{1/3} - x^{4/3}$$

2.09 $\quad -0.6$

① $f'(x) = 2\cdot\frac{1}{3}x^{-2/3} - \frac{4}{3}x^{1/3} = 0$, $f'(x) = \frac{2}{3x^{2/3}} - \frac{4}{3}x^{1/3}$

	-1000000	0	0.1	1/2	1000
f'	+		+		−
f	↗		↗		↘

$$\frac{2}{3}x^{-2/3} = \frac{4}{3}x^{1/3} \quad \Big|\cdot 3$$

$$2x^{-2/3} = 4x^{1/3}$$

$$\frac{2}{x^{2/3}} = 4x^{1/3} \quad \Big|\cdot x^{2/3}$$

$$2 = 4x$$

$\frac{1}{2} = x$, $x = 0$ ← because f' is undefined at $x=0$

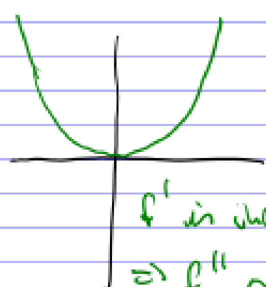
↳ is rel. max

Panel 6

What does f'' say about f? (f' tells if f incr/decr.)

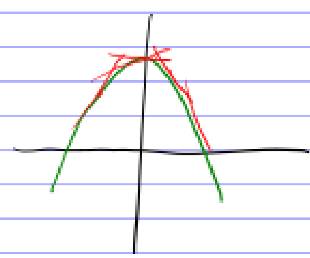
f'' is deriv. of f'

$\Rightarrow f''$ tells if f' incr/decr.!



f' is incr.

$\Rightarrow f''$ positive



f' is decr.

$\Rightarrow f''$ negative

6

Panel 7

Concavity: (which way road curves, which way to steer)

If $f''(x) > 0$ then f is concave up

If $f''(x) < 0$ then f is concave down

If $f''(c) = 0$ and f changes concavity at $x = c$: c is inflection point

Ex: $f(x) = x^4 - 4x^3$ - discuss concavity

$f'(x) = 4x^3 - 12x^2$

(1) $f''(x) = 12x^2 - 24x = 0$

$12x(x-2) = 0$

(2) $x = 0, x = 2$ possible infl. pts

(3)

	-100000	0	1	2	100000
f''	+		−		+
f	$\cup$		$\cap$		$\cup$

f is concave up on $(-\infty, 0)$ and $(2, \infty)$, concave down on $(0, 2)$

7

Panel 8

Ex: $f(x) = x^4 - 2x^2 + 3$ Discuss inc/dec/concavity

$f'(x) = 4x^3 - 4x = 4x(x^2 - 1) = 0$

$f''(x) = 12x^2 - 4 = 4(3x^2 - 1) = 0$

critical points: $x = 0, x = +1, x = -1$

poss. infl. pts: $x = \pm\sqrt{\frac{1}{3}}$

(1) Find f' and f''

(2) Set them equal to zero, (or where they don't exist)

(3) Make a big table with signs of f' and f''

		-1		$-1/\sqrt{3}$		0		$+1/\sqrt{3}$		1	
f'	−		+		+		−		−		+
f''	+		+		−		−		+		+
f											

$x = -1$ min

$x = -1/\sqrt{3}$: infl

$x = 0$: max

$x = \frac{1}{\sqrt{3}}$: infl

$x = +1$: min

8

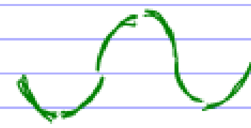
Panel 9

	-1		$-1/\sqrt{3}$		0		$1/\sqrt{3}$		1	
f'	$-$		$+$		$+$		$-$		$-$	$+$
f''	$+$		$+$		$-$		$-$		$+$	$+$
f										

It is now easy to find the graph:

$$0.114 - 0.666 + 3$$

$$f(x) = x^4 - 2x^2 + 3 = 2.4$$



	-1		$-1/\sqrt{3}$		0		$1/\sqrt{3}$		1	
f'	$-$		$+$		$+$		$-$		$-$	$+$
f''	$+$		$+$		$-$		$-$		$+$	$+$
f										

Eval f at all special points:

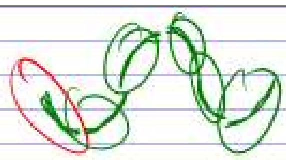
$f(-1) = 2$ $\quad f(1/\sqrt{3}) = 2.4$

$f(0) = 3$ $\quad f(-1/\sqrt{3}) = 2.4$

$f(1) = 2$

9

Panel 10

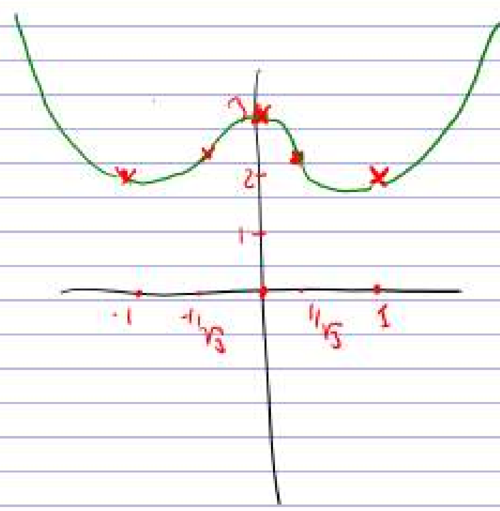


Eval f at all special points:

$f(-1) = 2$ $\quad f(1/\sqrt{3}) = 2.4$

$f(0) = 3$ $\quad f(-1/\sqrt{3}) = 2.4$

$f(1) = 2$



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