Panel 1

Differentiation

Ex: $f(x) = \sin(x^2)$

out, in

$\cos(x^2) \cdot 2x$

Power Rule $\frac{d}{dx}[x^p] = p x^{p-1}$

Product Rule $\frac{d}{dx}(f \cdot g) = f' \cdot g + f g'$

Quotient Rule $\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{f'g - fg'}{(g)^2}$

Chain Rule: $\frac{d}{dx}\left[f(g(x))\right] = f'(g(x)) \cdot g'(x)$

$\frac{d}{dx}\left[f\left(\text{what ever}\right)\right] = f'\left(\text{what ever}\right) \cdot \left(\text{inner deriv}\right)$

1

Panel 2

Chain Rule Examples

$f(x) = \sin(x^2)$ $f'(x) = \cos(x^2) \cdot 2x$

out, in

$f(x) = \sqrt{1-2x^2}$ $f'(x) = \frac{1}{2}(1-2x^2)^{-1/2} \cdot (-4x)$

$f(x) = \sin(x \cdot \cos(x^2))$ $f'(x) = \cos(x \cdot \cos(x^2)) \cdot [1 \cos(x^2) + x(-\sin(x^2) 2x)]$

$f(x) = \sin\left(\frac{x}{1-x}\right)$ $f'(x) = \cos\left(\frac{x}{1-x}\right) \cdot \left[\frac{1(1-x) + x}{(1-x)^2}\right]$

$f(x) = \left(\frac{\sin(x)}{\sin(1-x)}\right)^2$ $f'(x) = 2\left(\frac{\sin(x)}{\sin(1-x)}\right)^1 \cdot \left[\frac{\cos(x)\sin(1-x) + \cos(x)\cos(1-x)}{\sin^2(1-x)}\right]$

2

Panel 3

Quotient Rule: ~~canceled~~! replace by prod + chain!

$$\frac{f(x)}{g(x)} = f(x)\cdot[g(x)]^{-1}$$

$$f(x) = \tan(x) = \frac{\sin(x)}{\cos(x)} = \sin(x)\cdot[\cos(x)]^{-1}$$

$$f'(x) = \cancel{\cos(x)}\cdot\cancel{[\cos(x)]^{-1}} + \sin(x)\left(-[\cos(x)]^{-2}\cdot(-\sin(x)\right) =$$

$$1 + \frac{\sin^2(x)}{\cos^2(x)} = \frac{\cos^2(x)+\sin^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)} = \underline{\sec^2(x)}$$

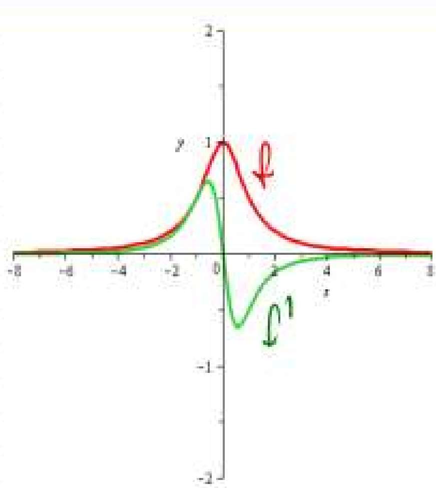
$$f(x) = \sin(\cos(\sin(\cos(x^2)))) \qquad f'(x) = \cos(\cos(\sin(\cos(x^2))))\cdot$$

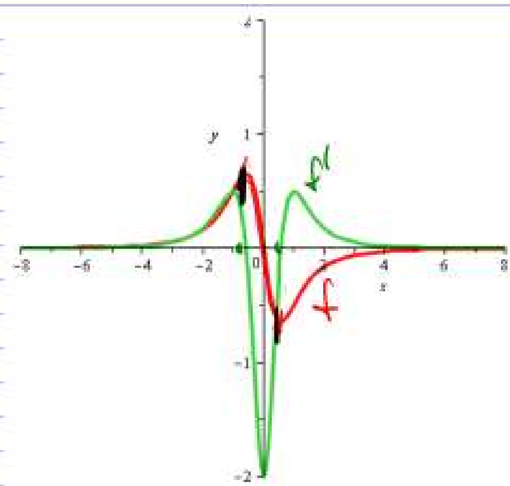
$$(-\sin(\sin(\cos(x^2)))\cdot\cos(\cos(x^2))(-\sin(x^2))\cdot 2x$$

3

Panel 4

Identify f and f'



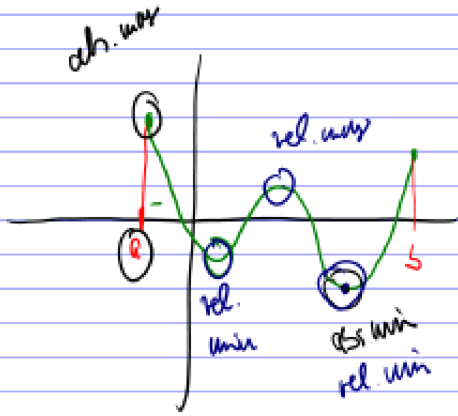


4

Panel 5

Applications of Derivatives

① Finding max/min



absolute max/min:
overall largest, smallest value of $f(x)$

local or relative max/min:
locally the largest, smallest value of $f(x)$

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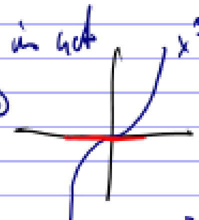
Panel 6

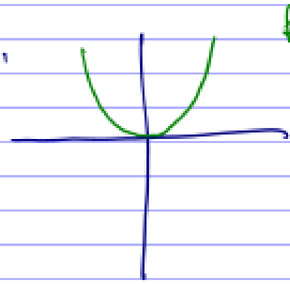
Thm: (Fermat's Theorem)

If f is diff'ble on (a,b) and f has a rel. max or min at $x=c$, then $f'(c)=0$. Converse is not true ⇒

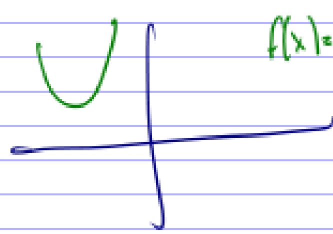


$f(x)=x^3$
$f'(x)=3x^2=0$
$x=0$

Ex:



$f(x)=x^2$
$f'(x)=2x=0 \Rightarrow x=0$ is a min!



$f(x)=ax^2+bx+c$ $\quad f'(x)=2ax+b=0$

$\boxed{x=-\frac{b}{2a}}$

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Panel 7

Recipe for finding relative extrema either max or min

Ex: $f(x) = x^3 - 3x^2 + 1$

$f'(x) = 3x^2 - 6x = 0$

$3x(x-2) = 0$

$x = 0, \; x = 2$

are critical points

	-1	0	1	2	10000000
f'	+		-		+
f	↗		↘		↗
	inc		dec		inc.

at $x = 0$, max

at $x = 2$, min

1. Take f'
2. Solve $f' = 0$
3. Critical points are $f'(x) = 0$ or $f'(x)$ dne.
4. check sign to the right/left of all critical points

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Panel 8

Thm: If f is diff'ble and $f'(x) > 0 \Rightarrow f$ goes up, increasing

If f is diff'ble and $f'(x) < 0 \Rightarrow f$ goes down, decreasing

Ex: $f(x) = x^{3/5}(4-x)$ Find local extrema

$$f'(x) = \frac{3}{5} x^{-2/5}(4-x) + x^{3/5}(-1) = 0$$

$$\frac{3}{5}\frac{4-x}{x^{2/5}} - x^{3/5} = \frac{3(4-x)}{5x^{2/5}} - \frac{5x}{5x^{2/5}} = \frac{12-8x}{5x^{2/5}} = 0$$

critical points: $x = \frac{12}{8} = \frac{3}{2}$, $x = 0$ (f' does not exist at 0)

	-1	0	1	3/2	5676739459
f'	+		+		-
f	↗		↗		↘

$x = 3/2$ is local (rel) max.!

($x = 0$ is nothing)

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Panel 9

Ex: Find local extrema for $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ HW
and identify all intervals where f is increasing

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