Panel 1

Review

Definitions:
- limit of f as x approaches a
- continuity of f at x=a
- derivative of f at x=a (2x)
- higher-order derivatives

Theorems:
- $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$ $\lim_{x\to 0} \frac{\cos(x)-1}{x} = 0$
- limit of sums, products, quotients
- Squeezing theorem,
- limits at infinity of rational functions
- continuity of polynom. + rational functions
- Intermediate Value theorem

1

Panel 2

Theorems
- differentiable implies continuity
- derivative of a sum
- derivative of a constant
- derivative of $c \cdot f(x)$
- Power rule
- Product rule
- Quotient rule

2

Panel 3

Limit: as x gets closer to a, f(x) gets closer to L.

If so: $\lim_{x \to a} f(x) = L$

(Given $\varepsilon > 0$ there is $\delta > 0$ s.t. if $|x-a| < \delta \Rightarrow |f(x) - L| < \varepsilon$)

To find limits:

(1) Plug in value

(2) $\frac{0}{0} \Rightarrow$ more work

$\lim_{x \to \pi/2} \frac{\sin(x)}{x}$

3

Panel 4

Continuity: f is continuous at x=a if

(1) f(a) exists

(2) $\lim_{x \to a} f(x)$ exists

(3) (1) = (2)

$\lim_{x \to a} f(x) = f(a)$

Graphically: If f has no holes or gaps.

Disconts: (a) removable

(b) jump

(c) essential

4

Panel 5

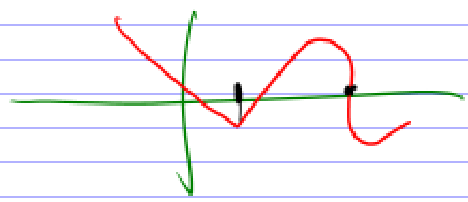
Derivative:

$$f'(x) = \frac{df}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{a \to x} \frac{f(a) - f(x)}{a - x} \quad \text{if exists}$$

Geometrically: slope of tangent

If f has a corner, kink, vert. tangent $\Rightarrow$ it is not diff'ble



5

Panel 6

Higher-Order Derivatives:

deriv. of deriv. (of deriv of deriv...)

$$f''(x) = \frac{d^2 f}{dx^2} = (f')'$$

6

Panel 7

Theorem (Limit of sums, products, and quotients)

as it should

7

Panel 8

Theorem: (Squeezing Theorem)

$$g(x) \le f(x) \le h(x)$$

$\Downarrow$ as $x \to a$

$$L$$

$$\lim_{x \to 0} x \sin\left(\frac{1}{x}\right) = 0$$

$$-x \le x \sin\left(\frac{1}{x}\right) \le x$$

$\Downarrow$ as $x \to 0$

$$0$$

8

Panel 9

Theorem: (Limit at infinity of rational functions)

$$\lim_{x \to \pm\infty} \frac{p(x)}{q(x)} = \begin{cases} \# & \text{if } \deg(p) = \deg(q) \\ 0 & \text{if } \deg(p) < \deg(q) \\ \begin{pmatrix} \text{not a number} \\ \pm\infty \end{pmatrix} & \text{if } \deg(p) > \deg(q) \end{cases}$$

9

Panel 10

Theorem: (Continuity of polynomials + rational functions)

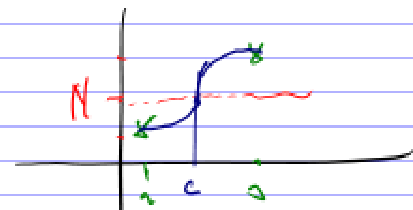
$$C(x) = \frac{x^2+7x-9}{x^2-4}$$ is cont. for all $x \neq \pm 2$

10

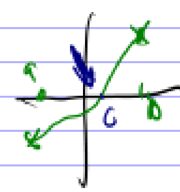
Panel 11

Theorem: (Intermediate Value Theorem)

General: if N is between $f(a)$ and $f(b)$, and f is cont. on $[a,b]$, then there is a $c \in (a,b)$ with $f(c) = N$



Check: If f is cont. on $[a,b]$ and $f(a) \cdot f(b) < 0$, i.e. $f(a)$, $f(b)$ have different signs then $f(c) = 0$ for some c



Panel 12

Theorem: (Differentiability implies Continuity) ✓

Panel 13

Theorem: (Derivatives of constants and $c \cdot f(x)$)

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}(c \cdot f(x)) = c \cdot f'(x)$$

13

Panel 14

Theorem (The Power Rule)

$$\frac{d}{dx}[x^p] = p\, x^{p-1}$$ for all p

14

Panel 15

Theorem (The Product and Quotient Rules)

$$\frac{d}{dx}(f \cdot g) = f' g + f \cdot g'$$

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{f' g - f g'}{(g)^2}$$

15

Panel 16

Theorem: (Derivatives of basic trig. functions)

$$\frac{d}{dx}\left[\sin(x)\right] = \cos(x)$$

$$\frac{d}{dx}\left[\cos(x)\right] = -\sin(x)$$

16

Panel 17

$$f(x)=\begin{cases}\frac{x^3-3x^2}{2x}, & \text{if } x\neq 0\\ 2, & \text{if } x=0\end{cases}$$

A) yes, cont. at x=0

B) No, it ain't

$$\lim_{x\to 0} f(x) = \lim_{x\to 0}\frac{x^3-3x^2}{2x} = \lim_{x\to 0}\frac{x^{\cancel{2}}(x-3)}{2\cancel{x}} = 0$$

17

Panel 18

Find the following limits at i...

$$\lim_{x\to\infty}\frac{2x+3x^4}{4x^3-2x^2+x-1}$$

A) 0

B) $\frac{3}{4}$

c) $+\infty$

d) $-\infty$

e) ?

18