Panel 1

All about Derivatives

Def.: $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$
$= \lim_{a \to x} \frac{f(a)-f(x)}{a-x} = \frac{df}{dx}$

Meaning: slope of tangent
(inst.) velocity
inst. rate of change

Power Rule:

$\frac{d}{dx}(x^n) = n x^{n-1}$

Constant Rule

$\frac{d}{dx}(c \cdot f(x)) = c \cdot \frac{d}{dx} f(x) = c \cdot f'(x)$

Product Rule

$\frac{d}{dx}(f \cdot g) = f' g + f \cdot g'$

Quotient Rule

$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{f' g - f g'}{(g)^2}$

Special derivatives: $\frac{d}{dx} \sin(x) = \cos(x)$, $\frac{d}{dx} \cos(x) = -\sin(x)$

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Panel 2

$f(x) = \pi^2 + x^2 + \sin(x) + \sqrt{x}$ $\quad - x^{1/2}$

$f'(x) = 2x + \cos(x) + \frac{1}{2} x^{1/2 - 1} = 2x + \cos(x) + \frac{1}{2} x^{-1/2}$

$f(x) = x^2 + \cos(x) + \frac{1}{x^3} + \sin(\pi^2)$ $\quad x^{-3}$

$f'(x) = 2x - \sin(x) - 3x^{-3-1} + 0 = 2x - \sin(x) - 3x^{-4}$

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Panel 3

$f(x)=x^2(x^4-2x)$ $= x^6 - 2x^3$, $f'(x) = 6x^5 - 6x^2$

$f'(x) = \underbrace{2x}(x^4-2x) + x^2 \cdot (\underbrace{4x^3-2}) = 6x^5 - 6x^2$

$f(x)=x^2\cos(x)$

$f'(x) = \underbrace{2x}\cos(x) + x^2\underbrace{(-\sin(x))}$

$2x\cos(x) - x^2\sin(x)$

$f(x)=x^3\sin(x)$

$f'(x) = 3x^2\sin(x) + x^3\underbrace{\cos(x)}$

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Panel 4

$f(x)=\cot(x)$ $= \frac{\cos(x)}{\sin(x)}$ $\Rightarrow$ $f'(x) = \frac{-\sin(x)\cdot\sin(x) - \cos(x)\cos(x)}{\sin^2(x)}$

$= \frac{-(\sin^2(x)+\cos^2(x))}{\sin^2(x)} =$

$= -1/\sin^2(x) = -\csc^2(x)$

$f(x)=\frac{\sin(x)}{x^4-3}$

$f'(x) = \frac{\cos(x)\cdot(x^4-3) - \sin(x)(\underline{4x^3})}{(x^4-3)^2}$

$\frac{d}{dx}\sin = \cos$

$\frac{d}{dx}\cos = -\sin$

$\frac{d}{dx}\tan = \sec^2(x)$

$\frac{d}{dx}\cot = -\csc^2$

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Panel 5

$$f(x)=\pi^2\sin\left(\frac{\pi}{6}\right)$$

$f'(x)=0$

$f(x)=\frac{x^4}{x^2}-\frac{2x}{x^2}+\frac{3}{x^2}=x^2-\frac{2}{x}+\frac{3}{x^2}$

$f'(x)=2x+2x^{-2}-6x^{-3}$

$$f(x)=\frac{x^4-2x+3}{x^2}$$

$f'(x)=\frac{(4x^3-2)(x^2)-(x^4-2x+3)(2x)}{(x^2)^2}$

$=\frac{4x^5-2x^2-2x^5+4x^2-6x}{x^4}$

$=\frac{2x^5+2x^2-6x}{x^4}=\frac{\cancel{x}(2x^4+2x-6)}{x^{\cancel{4}3}}$

$$f(x)=\frac{x^2}{x^4-2x+3}$$

$f'(x)=\frac{(2x)(x^4-2x+3)-x^2\cdot(4x^3-2)}{(x^4-2x+3)^2}$

Panel 6

$$f(x)=\frac{\sin(x)}{x^2-3\sqrt{x}}$$

$f'(x)=\frac{\cos(x)(x^2-3\sqrt{x})-\sin(x)(2x-3\frac{1}{2}x^{-1/2})}{(x^2-3\sqrt{x})^2}$

$$f(x)=\frac{(x)\sin(x)}{1-2x}$$

$f'(x)=\frac{(1\sin(x)+x\cos(x))(1-2x)-x\sin(x)(-2)}{(1-2x)^2}$

$f(x)=(\sin(x))\left(\frac{x}{\cos(x)}\right)$, $f'(x)=\cancel{\cos(x)}\cdot\frac{x}{\cancel{\cos(x)}}+\sin(x)\frac{(\cos(x)+x\sin(x))}{(\cos(x))^2}$

Panel 7

$$f(x) = \frac{x\sin(x)}{(x^2+1)\cdot\cos(x)}$$

$$f'(x) = \frac{(1\sin(x) + x\cdot\cos(x))\cdot(x^2+1)\cos(x) - (2x\cos(x) - (x^2+1)\sin(x))}{[(x^2+1)\cos(x)]^2}$$

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Panel 8

$f(x) = 3x^5 - 2x^3 + 5x - 1$, find $f''(x)$

$$f'(x) = 15x^4 - 6x^2 + 5$$

$$f''(x) = 60x^3 - 12x$$

$f(x) = 3x^5 - 2x^3 + 5x - 1$, find $f^{(7)}(x)$

$$f^{(7)}(x) = 0$$

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Panel 9

. $f(x) = \sin(x)$, find $f^{(24)}(x)$

$f'(x) = \cos(x)$

$f''(x) = -\sin(x)$

$f'''(x) = -\cos(x)$

$f^{(4)}(x) = +\sin(x)$

$f^{(24)}(x) = \sin(x)$

. $f(x) = x\sin(x)$, find $f'''(x)$

$f'(x) = \sin(x) + x\cos(x)$

$f''(x) = \cos(x) + \cos(x) - x\sin(x)$

$= 2\cos(x) - x\sin(x)$

$f'''(x) = -2\sin(x) - (\sin(x) + x\cos(x))$

$= -3\sin(x) - x\cos(x)$

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Panel 10

$$f(x) = \frac{\sin(x)\cdot\cos(x)}{1+\sqrt{x}}$$

$$f'(x) = \frac{(\cos^2(x) - \sin^2(x))(1+\sqrt{x}) + (\sin(x)\cos(x))(\frac{1}{2}x^{-1/2})}{(1+\sqrt{x})^2}$$

$$\frac{d}{dx}\sec(x) = \frac{d}{dx}[\cos(x)]^{-1} = \frac{d}{dx}\frac{1}{\cos(x)} = \frac{0\cdot\cos(x) + \sin(x)}{\cos^2(x)} = \frac{\sin(x)}{\cos(x)}\cdot\frac{1}{\cos(x)} = \tan\cdot\sec$$

$$\frac{d}{dx}\csc(x) = -\cot(x)\csc(x)$$

$$\frac{d}{dx}\frac{1}{\sin(x)} = \frac{-\cos(x)}{\sin^2(x)} = -\cot(x)\cdot\csc(x)$$

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Panel 11

Exam I next Wed

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