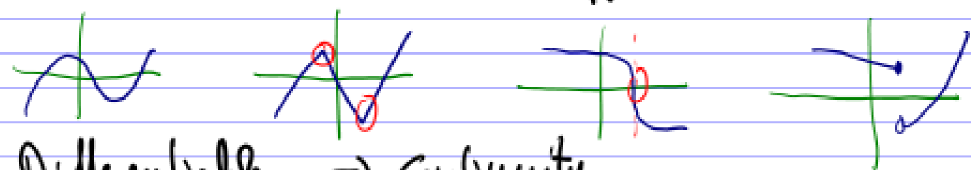
Panel 1

Last Time:

Alternative def's of derivative:

$$f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a} = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h} \quad \left(\text{change } a \text{ to } x\right)$$

Differentiable vs not differentiable



Differentiable ⇒ continuity

Proof:

$$\lim_{x\to a} f(x)-f(a) = \lim_{x\to a}\frac{f(x)-f(a)}{(x-a)}\cdot\underbrace{(x-a)}_{\lim_{x\to a}} = f'(a)\cdot 0 = 0$$

1

Panel 2

Differentiation Rules (1) : Power Rule

① $f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$

$$\frac{d}{dx}x^n = nx^{n-1}$$

derivative

Ex:

$f(x) = x^5 \Rightarrow f'(x) = 5x^4$

$f(x) = \frac{1}{x^4} = x^{-4} \Rightarrow f'(x) = -4x^{-5}$

$f(x) = \sqrt[3]{x^2} = x^{2/3} \Rightarrow f'(x) = \frac{2}{3}x^{\frac{2}{3}-1} = \frac{2}{3}x^{-1/3}$

$= \frac{2}{3\sqrt[3]{x}}$

2

Panel 3

Higher derivatives

$f(x) \Rightarrow f'(x) \Rightarrow (f'(x))' = f''(x)$ 2nd deriv

$f'''(x)$ 3rd deriv

$\frac{d}{dx} f = f'$

$f^{(4)}(x)$ 4th deriv

$\frac{d^2}{dx^2} f = f''$

$\Rightarrow \frac{d^n}{dx^n} f(x) = f^{(n)}(x)$ n^{th} deriv

$f(x) = \frac{1}{x^4}$. Find f''

$f(x) = x^{-4}$, $f'(x) = -4x^{-5} = -\frac{4}{x^5}$, $f''(x) = (-4)\cdot(-5)x^{-6} = 20x^{-6}$

3

Panel 4

Differentiation Rules (2): Constant Rule

$\frac{d}{dx} c \cdot g(x) = c \frac{d}{dx} g(x)$

$\frac{d}{dx} C = 0$

Ex

$f(x) = 5x^2 \Rightarrow f'(x) = 5 \cdot 2x = 10x$

$g(x) = -3/x^2 = -3x^{-2}$ $g'(x) = 6x^{-3}$

$h(x) = \frac{5}{9} \sqrt[3]{x^2} = \frac{5}{9} x^{2/3}$ $h'(x) = \frac{5}{9} \cdot \frac{2}{3} x^{-1/3}$

$k(x) = 2^3$ (constant) $k'(x) = 0$

4

Panel 5

Differentiation Rules (3): Add / Subtract Rule

$$\frac{d}{dx}\left(f(x) \pm g(x)\right) = \frac{d}{dx} f(x) + \frac{d}{dx} g(x)$$

Ex: $f(x) = x^2 + x^5 - \frac{1}{x}$ ~ x^{-1}

$\Rightarrow f'(x) = 2x + 5x^4 + x^{-2}$

$g(x) = 3x^2 - 2\sqrt{x} + 9/x^3 - \sqrt{2}$, Find g'' ($\sqrt{x} = x^{1/2}$)

$g'(x) = 6x - 2\frac{1}{2}x^{-1/2} + 9(-3)x^{-4} + 0$

$$\boxed{g''(x) = 6 - 2\frac{1}{2}\left(-\frac{1}{2}\right)x^{-3/2} + 9\cdot(-3)(-4)x^{-5}}$$

5

Panel 6

Ex: Find points for $y = x^4 - 6x^2 + 4$ where the tangent line is horizontal.

$\Rightarrow$ slope of tangent is zero!

$y' = 0$

$y' = 4x^3 - 12x = 0$

$4x(x^2 - 3) = 0$

$\Rightarrow x = 0$, $x = \sqrt{3}$, $x = -\sqrt{3}$

6

Panel 7

Ex: Position of particle is $s(t) = t^3 - 6t^2 + 9t$

a) velocity at time t

b) when is particle at rest?

c) when is particle moving forward?

d) acceleration at time t $\quad a(t) = 6t - 12$

$[9t^1]' = 1 \cdot 9x^0 = 9$

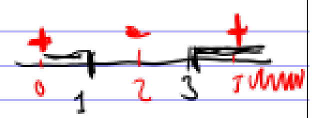
d) $a(t) = v'(t) = s''(t)$

a) $s'(t) = 3t^2 - 12t + 9$

b) $0 = s'(t) = 3(t^2 - 4t + 3) = 3(t-3)(t-1) + 0$

$t = 1, t = 3$

c) $s'(t) > 0 \qquad 3(t-3)(t-1) > 0$



$t \in [0,1) \cup (3, \infty)$

t is time, can't be negative

Panel 8

Differentiation Rules (4): Product

$$\frac{d}{dx}\big(f(x) \cdot g(x)\big) = \cancel{\left(\frac{d}{dx}f(x)\right)\left(\frac{d}{dx}g(x)\right)}$$

Ex: $f(x) = x^5 \quad , f'(x) = 5x^4$ ✓

$= (x^2)(x^3) \quad f'(x) = (2x)(3x^2) = 6x^3$???

Product rule is more complicated.

$$\frac{d}{dx}\big(f(x) \cdot g(x)\big) = f'(x)\,g(x) + f(x)\,g'(x)$$

Ex: $h(x) = \underbrace{x^2}_{f} \cdot \underbrace{x^3}_{g} \Rightarrow h'(x) = 2x \cdot x^3 + x^2 \cdot 3x^2 = 5x^4$ ✓

Panel 9

Ex: $f(x) = x^2 (2x-3)$

$f'(x) = \underbrace{2x}(2x-3) + x^2 \underbrace{2}$

$f(x) = (x^2 - 2x)(3x^2 + 9x - 1)$

$f'(x) = (2x-2)(3x^2+9x-1) + (x^2-2x)(6x+9)$

9

Panel 10

Ex: $f(x) = x^2 \cdot \sin(x)$

$f'(x) = \underbrace{2x}\sin(x) + x^2 \cdot \cos(x)$

$\frac{d}{dx}\sin(x) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h\to 0} \frac{\sin(a+h) - \sin(a)}{h} =$

$= \lim_{h\to 0} \frac{\sin(a)\cos(h) + \cos(a)\sin(h) - \sin(a)}{h} =$

$= \lim_{h\to 0} \frac{\sin(a)\cos(h) - \sin(a)}{h} + \lim_{h\to 0} \frac{\cos(a)\sin(h)}{h}$ $\to 1$

$= \lim_{h\to 0} \sin(a)\frac{\cos(h) - 1}{h} + \quad \cos(a)$ $\to 0$

$= \cos(a)$

10

Panel 11

Important

$$\frac{d}{dx}\sin(x) = \cos(x)$$

$$\frac{d}{dx}\cos(x) = -\sin(x)$$

11

Panel 12

What is $\frac{d}{dx}\cos(x)$?

12

Panel 13

Differentiation Rules (5): Quotient Rule!

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{f'(x)\,g(x) - f(x)\cdot g'(x)}{[g(x)]^2}$$

$$\left(\frac{hi}{ho}\right)' = \frac{ho\,dhi - hi\,dho}{(ho)^2}$$

don't tell I found this cute!

Ex: $f(x) = \dfrac{x^2}{\sin(x)}$

$$f'(x) = \frac{2x\cdot\sin(x) - x^2\cdot\cos(x)}{[\sin(x)]^2}$$

13