Panel 1

Math 1501: Last time

Limits at Infinity: $\lim\limits_{x\to\pm\infty} \frac{p(x)}{q(x)} = \begin{cases} \frac{\#}{\#} & \text{if } \deg(p)=\deg(q) \\ 0 & \text{if } \deg(p)<\deg(q) \\ \pm\infty & \text{else} \end{cases}$

Differentiation: $f'(a) = \lim\limits_{x\to a} \frac{f(x)-f(a)}{x-a}$

Meaning: "slope of tangent"

Shortcut rule for $f(x) = x^n$:

1

Panel 2

Use ~~def. of deriv.~~ to find f' for $f(x) = 2x^2 - x$

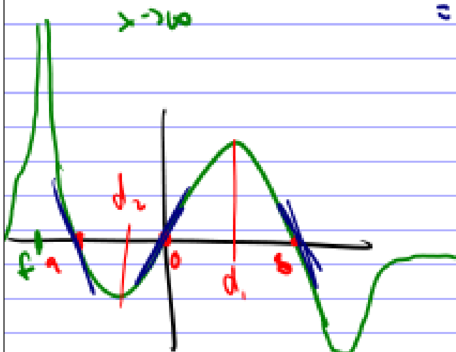
$$f'(x_0) = \lim_{x\to x_0} \frac{f(x_0)-f(x)}{x_0-x} = \lim \frac{(2x_0^2 - x_0) - (2x^2 - x)}{x_0 - x} =$$

$$= \lim \frac{2(x_0^2 - x^2) - (x_0 - x)}{x_0 - x}$$

$$= \lim_{x\to x_0} \frac{2(x_0+x)\cancel{(x_0-x)} - \overset{1}{\cancel{(x_0-x)}}}{\cancel{(x_0-x)}} = 2x_0 - 1$$

$\lim\limits_{x\to f^-} f(x) = \infty$

$\lim\limits_{x\to\infty} f(x) = 0$



$f'(a) < 0$ $\quad f(a) = 0$

$f'(c) > 0$

$f'(b) < 0$

$f'(d_1) = f'(d_2) = 0$

2

Panel 3

Note: $f'(x_0) = \lim\limits_{x \to x_0} \dfrac{f(x_0) - f(x)}{x_0 - x} =$

$= \lim\limits_{x \to x_0} \dfrac{f(x) - f(x_0)}{x - x_0} =$

$f'(a) = \lim\limits_{x \to a} \dfrac{f(a) - f(x)}{a - x} =$

$= \lim\limits_{x \to a} \dfrac{f(x) - f(a)}{x - a}$

let $x - a = h$ $\quad = \lim\limits_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$

3

Panel 4

$\lim\limits_{u \to \infty} \dfrac{4u^4 + 5}{(u^2 - 2)(2u^2 - 1)}$ $\quad \lim\limits_{u \to \infty} \dfrac{4u^4 + \dots}{2u^4 \dots} = 2$

4 5 6

$\lim\limits_{x \to -\infty} \dfrac{3x^{\cancel{4}} - 7x + 9}{5x^4 - 7x^3 + 9x - 1} = 0$

$= 3/5$

$= -\infty$

$= +\infty$

$\dfrac{3x^5}{5x^4} \sim \dfrac{3}{5}x$

$\dfrac{3x^6}{5x^4} \sim \dfrac{3}{5}x^2$

4

Panel 5

14) $s = t^2 - 8t + 18$, $s'(t) = 2t - 8$

avg. velocity $[3,4]$: $\frac{s(4)-s(3)}{4-3} = \frac{+2-3}{1} = \underline{\underline{-1}}$

inst. vel at 3: $s'(3) = 6-8 = -2$

at 4: $s'(4) = 8-8 = 0$

Use def. of deriv to find f' if $f(x) = 7x^2 - 2$

$$\lim_{x \to x_0} \frac{f(x_0) - f(x)}{x_0 - x} = \lim_{x \to x_0} \frac{(7x_0^2 - 2) - (7x^2 - 2)}{x_0 - x} = \frac{7(x_0+x)\cancel{(x_0-x)}}{\cancel{x_0-x}} = \underline{14x_0}$$

$$\lim_{x \to a} \frac{f(a) - f(x)}{a - x}$$

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \frac{(7(a+h)^2 - \cancel{2}) - (7a^2 - \cancel{2})}{h}$$

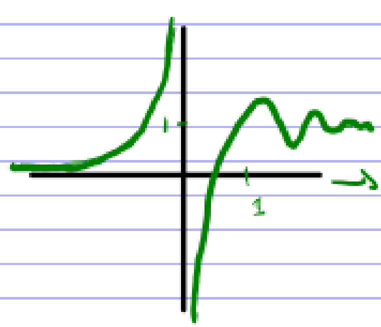
$$= \frac{\cancel{7a^2} + 14ah + 7h^2 - \cancel{7a^2}}{h} = \lim_{h \to 0} \frac{\cancel{h}(14a + 7h)}{\cancel{h}} = 14a$$

Panel 6

Quiz #4

Name: ____________

① Find the indicated limits.



a) $\lim_{x \to \infty} f(x) = 1$

b) $\lim_{x \to -\infty} f(x) = 0$

c) $\lim_{x \to 0^-} f(x) = +\infty$

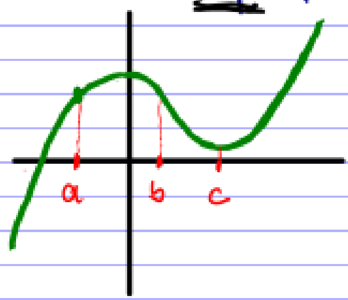
② a) $\lim_{x \to \infty} \frac{3x^4 - 2x + 1}{1 - x^4} = (-)3$

b) $\lim_{x \to -\infty} \frac{x^3 - x}{x^4 - x^2 + 1} = 0$

6

Panel 7

(3) Find the sign of the derivative f' at the given point.



a) $f'(a) > 0$

b) $f'(b) < 0$

c) $f'(c) \approx 0$

(4) Use the definition of derivative to find $f'(x)$ for

$f(x) = x^2 + 3$

$$\lim_{x \to x_0} \frac{f(x_0) - f(x)}{x_0 - x} = \frac{x_0^2 + \cancel{3} - (x^2 + \cancel{3})}{x_0 - x}$$

$f'(x) = 2x$

$$\lim_{x \to x_0} = \frac{(x_0 + x)\cancel{(x_0 - x)}}{\cancel{x_0 - x}} = 2x_0$$

7

Panel 8

Alternate form of derivative:

$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} =$$

$$= \lim_{x \to x_0} \frac{f(x_0) - f(x)}{x_0 - x} =$$

$$= \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

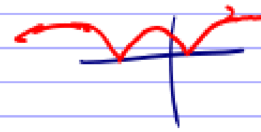
Subst: $x_0 - x = h$

8

Panel 9

Not every function has a derivative.

Ex: $f(x) = |x|$. Does f have derivative at $a = 0$:



$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{|h|}{h} \quad \begin{cases} \lim_{h \to 0^+} \frac{h}{h} = 1 \\ \lim_{h \to 0^-} \frac{-h}{h} = -1 \end{cases}$$

d.n.e.

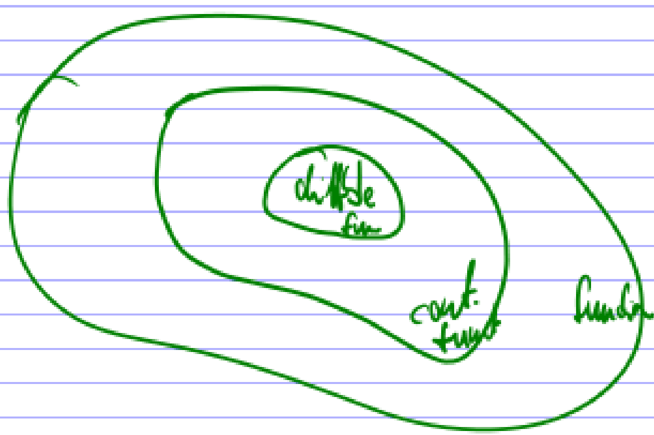
If f does not have a derivative at $x=a$ it is called not differentiable. Graphically it has a kink or corner!

9

Panel 10

Theorem: If $f(x)$ is differentiable at $x=a$ then $f(x)$ is continuous at $x=a$. The converse is false

$f(x)=|x|$



10

Panel 11

Short cuts for differentiation: power rule

(1) $f(x) = x^{n} \Rightarrow f'(x) = n x^{n-1}$

Ex: $f(x) = x^{5}$

$f(x) = \frac{1}{x^{4}}$

$f(x) = \sqrt[3]{x^{2}}$

HW

11