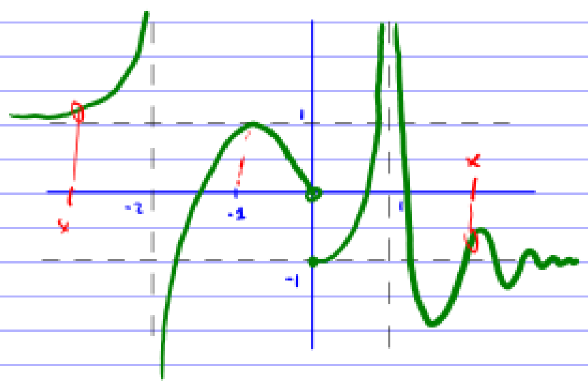
Panel 1

Last Time:



$\lim_{x\to\infty} f(x) = -1$

$\lim_{x\to -\infty} f(x) = 1$

$\lim_{x\to -1} f(x) = 1$

$\lim_{x\to -1^-} f(x) = 1$

$\lim_{x\to -2^-} f(x) = \infty$ $\qquad \lim_{x\to -2^+} f(x) = -\infty$ $\qquad \lim_{x\to -2} f(x)$ dne

$\lim_{x\to 1^-} f(x) = \infty$ $\qquad \lim_{x\to 1^+} f(x)\ \infty$ $\qquad \lim_{x\to 1} f(x)\ \infty$

$\lim_{x\to 0^-} f(x) = 0$ $\qquad \lim_{x\to 0^+} f(x) -1$ $\qquad \lim_{x\to 0} f(x)$ dne

Panel 2

Algebraic limits at infinity (via shortcut)

$$\lim_{x\to\infty} \frac{2-5x^2}{7x^2-2x-1} = -5/7$$

$$\lim_{x\to -\infty} \frac{5x^3-7x}{x^4+9x^3-2} \sim \frac{5}{x} = 0$$

$$\lim_{x\to\infty} \frac{1-x-x^2-x^3}{2+2x+2x^2} \sim -\frac{1}{2}x = -\infty$$

Panel 3

$$\lim_{x\to\infty}\left(\sqrt{x^2+9}-x\right)\cdot\frac{\sqrt{x^2+9}+x}{\sqrt{x^2+9}+x} \quad \lim_{x\to\infty}\frac{(\cancel{x^2}+9)-\cancel{x^2}}{\sqrt{x^2+9}+x}=0$$

$$\sim \sqrt{x^2}-x\approx 0$$

$$\lim_{x\to\infty}\sqrt{x^5+9}-x=+\infty$$

$$\sim x^{5/2}-x$$

$$\lim_{x\to\infty}\sqrt[5]{x^2+9}-x=-\infty$$

$$\sim x^{2/5}-x$$

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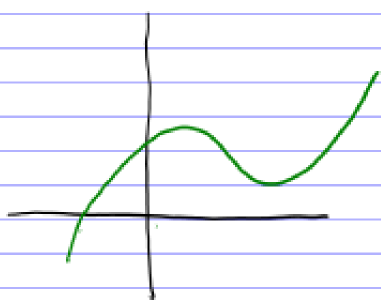
Panel 4

Next: Differentiation



Goal: Investigate rates of change.

Ex: $h(t)=4.9t^2$ is height of a ball dropped from observation deck.

Rate of change: $\frac{\text{height}}{\text{time}}=\frac{\text{distance}}{\text{time}}=$ speed.

a) Average speed after 5 seconds: $\frac{h(5)-h(0)}{5-0}=\frac{4.9\cdot 5^2-0}{5}=24.5$

b) Actual speed at 5 seconds: $\lim_{t\to 5}\frac{h(5)-h(t)}{5-t}=\lim_{t\to 5}\frac{122.5-4.9t^2}{5-t}$

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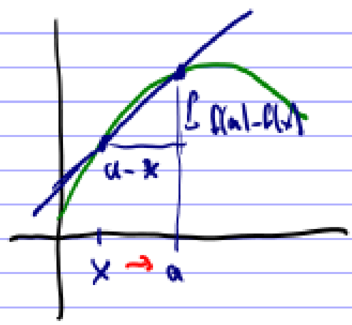
Panel 5

$$\lim_{t\to 5}\frac{h(5)-h(t)}{5-t} = \lim_{t\to 5}\frac{4.9(25-t^2)}{5-t} = \lim_{t\to 5}\frac{4.9(5+t)\cancel{(5-t)}}{\cancel{(5-t)}} =$$

$$= \underline{\underline{49}}$$

is inst. rate of change!



Def: $\lim_{x\to a}\dfrac{f(a)-f(x)}{a-x}$

is the derivative of $f(x)$ at $x=a$

I write this as $f'(a)$ or $\dfrac{d}{dx}f(x)\Big|_{x=a}$

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Panel 6

Ex: $f(x)=x^2$, find $f'(2)$, $\dfrac{d}{dx}f(x)\Big|_{x=-2}$, $f'(3)$, $f'(-5)$

$$f'(2)=\lim_{x\to 2}\frac{f(2)-f(x)}{2-x}=\lim_{x\to 2}\frac{4-x^2}{2-x}=\lim_{x\to 2}(x+2)=\underline{\underline{4}}$$

$$\frac{d}{dx}f(x)\Big|_{x=-2}=\lim_{x\to -2}\frac{f(-2)-f(x)}{-2-x}=\lim_{x\to -2}\frac{4-x^2}{-2-x}=\lim_{x\to -2}\frac{(2-x)\cancel{(2+x)}}{-\cancel{(2+x)}}$$

$$=-4$$

Instead of doing this over again:

$$\underline{\underline{f'(x_0)}}=\lim_{x\to x_0}\frac{f(x_0)-f(x)}{x_0-x}=\lim_{x\to x_0}\frac{x_0^2-x^2}{x_0-x}=\lim_{x\to x_0}\frac{(x_0+x)\cancel{(x_0-x)}}{\cancel{x_0-x}}$$

$$=\lim_{x\to x_0}(x_0+x)=\underline{\underline{2x_0}} \quad\Rightarrow\quad f'(3)=2\cdot 3=\underline{\underline{6}}$$

$$f'(-5)=\underline{\underline{-10}}$$

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Panel 7

Ex: $f(x) = x^3$, find $f'(x)$

Def: $f'(x) = \lim_{x_0 \to x} \frac{f(x) - f(x_0)}{x - x_0}$ is derivative of f at x

$f'(a) = \lim_{x \to a} \frac{f(a) - f(x)}{a - x}$

$f(x) = x^3$: $f'(x) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} = \lim_{x \to x_0} \frac{x^3 - x_0^3}{x - x_0}$.

$= \lim_{x \to x_0} \frac{\cancel{(x - x_0)}(x^2 + xx_0 + x_0^2)}{\cancel{x - x_0}} =$

$= \lim_{x \to x_0} x^2 + xx_0 + x_0^2 = \boxed{3x_0^2}$

$\Rightarrow \boxed{f(x) = x^3}$ then $\boxed{f'(x) = 3x^2}$

Panel 8

Ex: If $f(x) = \frac{1}{x}$, find $f'(-2)$

$\lim_{x \to -2} \frac{f(-2) - f(x)}{-2 - x}$

$\lim_{x \to x_0} \frac{f(x_0) - f(x)}{x_0 - x} = \lim_{x \to x_0} \frac{\frac{1}{x_0} - \frac{1}{x}}{x_0 - x} =$

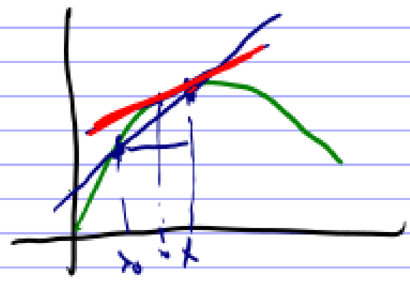
$= \lim_{x \to x_0} \frac{\frac{x - x_0}{x_0 x}}{x_0 - x} = \lim_{x \to x_0} \frac{\cancel{x - x_0}}{x_0 x} \cdot \frac{-1}{\cancel{x_0 - x}} =$

$= -\lim_{x \to x_0} \frac{1}{x x_0} = -\frac{1}{x_0^2}$

If $f(x) = \frac{1}{x}$ then $f'(x) = -\frac{1}{x^2}$

Panel 9

What is this derivative thing



$\dfrac{f(x)-f(x_0)}{x-x_0}$ is slope of line

as $x \to x_0$, that line turns into "tangent" line.

Thus $f'(x)$ is slope of tangent line

$f'(x)$ is the instantaneous rate of change

If f is distance, t is time: $f'(t)$ is inst. speed or velocity

all are Derivatives

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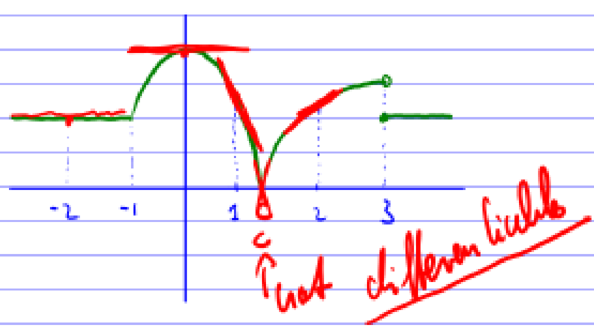
Panel 10

Ex. Consider the following graph of a function f and find the indicated derivatives:



$f'(0) = 0$

$f'(-2) = 0$

$f'(1) < 0$

$f'(2) > 0$

$f'(3)$ d.n.e.

1) where is f not continuous? at $x=3$

Where is f not differentiable? at $x=3$ and $x=c$!

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Panel 11

Short cuts for differentiation:

$f(x) = x^2 \Rightarrow f'(x) = 2x$

$f(x) = x^3 \Rightarrow f'(x) = 3x^2$

$f(x) = x^4 \Rightarrow f'(x) = 4x^3$

$f(x) = x^5 \Rightarrow f'(x) = 5x^{-4}$

$f(x) = \frac{1}{x} = x^{-1} \Rightarrow f'(x) = -x^{-2}$

$f(x) = \sqrt{x} = x^{1/2} \Rightarrow f'(x) = \frac{1}{2}x^{-1/2}$ ← check

If $f(x) = x^n$, $f'(x) = nx^{n-1}$

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Panel 12

$$f'(x) = \lim_{x \to x_0} \frac{\sqrt{x} - \sqrt{x_0}}{x - x_0} \cdot \frac{(\sqrt{x} + \sqrt{x_0})}{(\sqrt{x} + \sqrt{x_0})} =$$ if $f(x) = \sqrt{x}$

$$= \lim_{x \to x_0} \frac{\cancel{x - x_0}}{\cancel{x - x_0}} \cdot \frac{1}{(\sqrt{x} + \sqrt{x_0})} = \frac{1}{2\sqrt{x_0}} = \frac{1}{2}x_0^{-1/2}$$

$f(x) = \sqrt[5]{x^9} = x^{9/5}$, $f'(x) = \frac{9}{5}x^{4/5}$

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