Panel 1

Math 1401: Last time

Continuity: $\lim_{x \to a} f(x) = f(a)$ — $f(a)$ ✓ ; $\lim_{x \to a} f(x)$ ✓ ; " = "

Graphical meaning: gap or hole ⇒ not cont

$$f(x) = \begin{cases} \frac{x-2}{x^2-4} & x \neq 2 \\ c & x = 2 \end{cases}$$

Types of Discontinuities: removable, jump, essential

$$\lim_{x \to 2} \frac{x-2}{x^2-4} = \lim_{x \to 2} \frac{1}{x+2} = \frac{1}{4}$$

$c = 1/4$

Limits to infinity: $\lim_{x \to a^{\pm}} f(x) = \pm\infty$ vertical asymptote

1

Panel 2

$$f(x) = \begin{cases} cx & \text{if } x < 1 \\ x^2 - 9 & \text{if } 1 \leq x \leq 2 \\ 7 & \text{if } x > 2 \end{cases}$$

Where is f cont.?

$\lim_{x \to 1} f(x)$

$\lim_{x \to 1^-} cx = c$

$\lim_{x \to 1^+} x^2 - 9 = -8$

Take $c = -8$, then f is cont at $x = 1$

$\lim_{x \to 2} f(x) =$

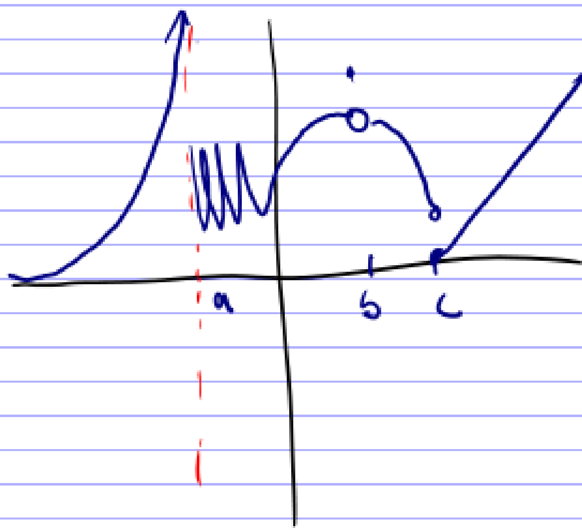
$\lim_{x \to 2^+} 7 = 7$

$\lim_{x \to 2^-} x^2 - 9 = -5$

Not cont. at $x = 2$, in fact, it is a jump

2

Panel 3



at a: essential

at b: removable

at c: jump

$$\lim_{x \to a^-} f(x) = \infty$$

$$\lim_{x \to c^+} f(x) = 0$$

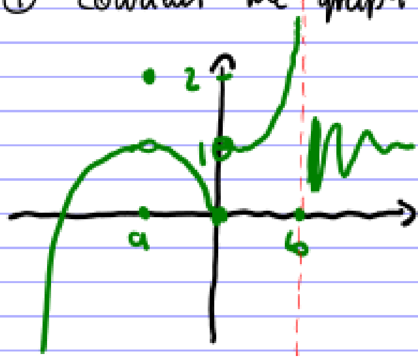
3

Panel 4

Name: ______________

Quiz #3a

① Consider the graph below.



a) $\lim_{x \to a} f(x) = 1$

b) $\lim_{x \to 0^-} f(x) = 0$

c) $\lim_{x \to 6^-} f(x) = \infty$

c) is f cont. at $x = a$? If not, what type: rem.

d) is f cont. at $x = 0$? If not, what type: jump

e) is f cont. at $x = 6$? If not, what type: ess.

4

Panel 5

(2) Where is the following function continuous?

$$f(x) = \frac{\sqrt{x-1}}{x^2-5x+6}$$

$(x-2)(x-3)$

$x \geq 1$ except $x = 2, 3$

$[1,2)(2,3)(3,\infty)$

(3) If $g(x) = \begin{cases} cx-1 & , \text{ if } x \leq 1 \\ x^2 & , \text{ if } x > 1 \end{cases}$

Find a value for c, if any, so that f is continuous at x=1

$\lim_{x\to 1^-} cx-1 = c-1$

$\lim_{x\to 1^+} x^2 = 1$

$\Big\} \; c-1=1$

$\underline{\underline{c=2}}$

5

Panel 6

2. For the function g whose graph is given, state the following

(a) $\lim_{x\to\infty} g(x)$ (b) $\lim_{x\to-\infty} g(x)$ later

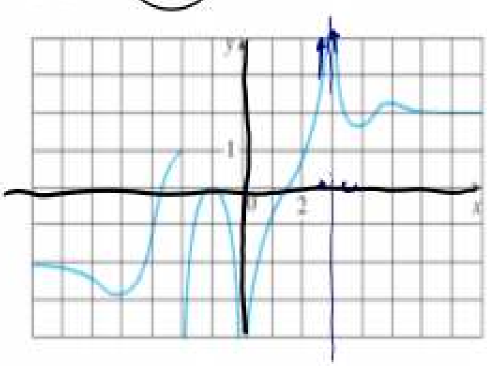
(c) $\lim_{x\to 3} g(x) = \infty$ (d) $\lim_{x\to 0} g(x)$ $-\infty$

(e) $\lim_{x\to -2^+} g(x)$ $-\infty$ (f) The equations of the asymptotes

$x=3$

$x=0$

$x=-2$



$\lim_{x\to -2^-} f(x) = 1$

$\lim_{x\to -2} f(x) = \underline{d.n.e.}$

6

Panel 7

$$\left.\begin{array}{l} \lim\limits_{x\to 1^+} \dfrac{1}{x^3-1} = +\infty \\[2ex] \lim\limits_{x\to 1^-} \dfrac{1}{x^3-1} = -\infty \end{array}\right\} \quad \lim_{x\to 1} f(x) = \text{d.n.e}$$

$$\left.\begin{array}{l} \lim\limits_{x\to 1^+} \dfrac{1}{(x-1)^3} = +\infty \\[2ex] \lim\limits_{x\to 1^-} \dfrac{1}{(x-1)^3} = -\infty \end{array}\right\} \quad \lim_{x\to 1} = \text{d.n.e}$$

$$\lim_{x\to 1} = +\infty$$

$$\lim_{x\to 1^+} \frac{1}{(x-1)^4} = +\infty \qquad \lim_{x\to 1^-} \frac{1}{(x-1)^4} = +\infty$$

7

Panel 8

Def: Let f be defined in $[a,\infty)$. Then

$$\lim_{x\to\infty} f(x) = L$$

means that $f(x)$ gets close to L if x gets ever larger.

$$\lim_{x\to\infty} \frac{1}{x} = 0 \qquad \lim_{x\to-\infty} \frac{1}{x} = \cancel{-0} = 0$$

$$\lim_{x\to\infty} x^3 = \infty \qquad \lim_{x\to-\infty} x^3 = -\infty$$

$$\lim_{x\to-\infty} x^4 = +\infty$$

8

Panel 9

Algebraic limits at infinity

Factor highest powers!

$$\lim_{x\to\infty} \frac{3x^2-x-2}{5x^2+4x+1} = \lim_{x\to\infty} \frac{\cancel{x^2}\left(3-\overset{\to 0}{\cancel{\frac{1}{x}}}-\overset{\to 0}{\cancel{\frac{2}{x^2}}}\right)}{\cancel{x^2}\left(5+\underset{\to 0}{\cancel{\frac{4}{x}}}+\underset{\to 0}{\frac{1}{x^2}}\right)} = \frac{3}{5}$$

$$\lim_{x\to -\infty} \frac{3x^2-7}{5x^4-3x^2+9} = \lim_{x\to -\infty} \frac{\cancel{x^2}\left(3-\cancel{\frac{7}{x^2}}\right)}{\cancel{x^4}\left(5-\cancel{\frac{3}{x^2}}+\cancel{\frac{9}{x^4}}\right)} \quad \frac{3}{5x^2} = 0$$

$$\lim_{x\to -\infty} \frac{5x^3-7x^2+9}{4x+7} = \lim_{x\to -\infty} \frac{x^3\left(5-\overset{\to 0}{\cancel{\frac{7}{x}}}+\overset{\to 0}{\cancel{\frac{9}{x^3}}}\right)}{x\left(4+\underset{\to 0}{\cancel{\frac{7}{x}}}\right)} \quad \frac{5x^2}{4} = \infty$$

Panel 10

Algebraic limits at infinity: Shortcut Rules for limits at infinity:

$$\lim_{x\to\infty} \frac{x^2-7x+9}{8-2x^2} = -\frac{1}{2}$$

$$\lim_{x\to\infty} \frac{x^3-7x+7}{5x^4-9x} = 0$$

$$\lim_{x\to\infty} \frac{5-x^4}{x^3-2} = \pm\infty = -\infty$$

$$\lim_{x\to -\infty} \frac{5-x^4}{x^3-2} = \cancel{\pm\infty}\ +\infty$$

$$\lim_{x\to -\infty} \frac{5-\cancel{x^4}}{\cancel{x^3}-2} = -x^2 = +\infty$$

⇒ if deg(top) < deg(bottom):

⇒ 0

⇒ if deg(top) > deg(bottom):

⇒ $\pm\infty$

⇒ if deg(top) = deg(bottom):

⇒ #/#

Panel 11

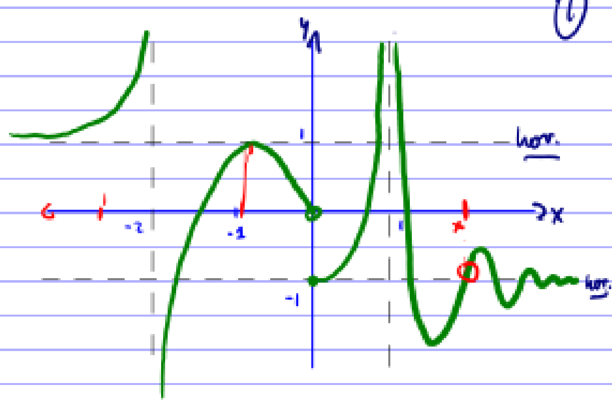
$$\lim_{x\to\infty} \frac{7x^3-5x^4+7x-9}{3x-3x^2-3x^3-3x^4} = \frac{-5}{-3} = \frac{5}{3}$$

$$\lim_{x\to\infty} \frac{7x^3+7x-9}{1-x^5} = 0$$

11

Panel 12

Limits Graphically



① a) $\lim_{x\to\infty} f(x) = -1$

b) $\lim_{x\to-\infty} f(x) = 1$

c) $\lim_{x\to-1} f(x) = 1$

d) $\lim_{x\to-1^-} f(x) = 1$

② a) $\lim_{x\to-2^-} f(x)$ b) $\lim_{x\to-2^+} f(x)$ c) $\lim_{x\to-2} f(x)$

③ a) $\lim_{x\to1^-} f(x)$ b) $\lim_{x\to1^+} f(x)$ c) $\lim_{x\to1} f(x)$

④ a) $\lim_{x\to0^-} f(x)$ b) $\lim_{x\to0^+} f(x)$ c) $\lim_{x\to0} f(x)$

12