Panel 1

Math 1401: Last time

Continuity: f is cont. at $x=a$: $\lim_{x\to a} f(x) = f(a)$ $\left\{ \begin{array}{l} f(a) \checkmark \\ \lim_{x\to a} f(x) \checkmark \\ \text{equal} \end{array} \right.$

Graphical meaning: no holes or gaps → cont.

Types of Discontinuities: removable: $\lim_{x\to a} f(x)$ exists
jump: $\lim_{x\to a^+}$, $\lim_{x\to a^-}$ exist
essential: otherwise

Intermediate Value Theorem:
If $f(a) < N < f(b) \Rightarrow$ there is $c \in (a,b)$ with $f(c) = N$, if f is cont.

Application of IVT: If $f(a)$, $f(b)$ have different signs, $f(c)=0$

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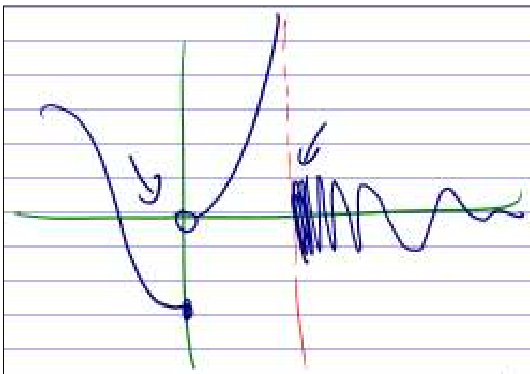
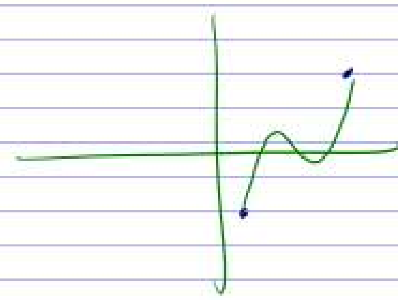
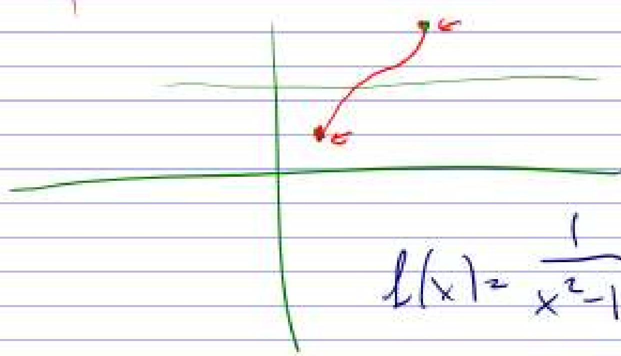
Panel 2







$f(x) = \frac{1}{x^2-1}$

$f(x) = \sqrt{x-2}$

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Panel 3

$$f(x) = \begin{cases} \sin(x), & x < \pi/4 \\ c, & x = \pi/4 \\ \cos(x), & x > \pi/4 \end{cases}$$

What is c s.t. f is cont. at $x = \pi/4$

$$\lim_{x \to \pi/4^-} \sin(x) = \sqrt{2}/2 = \lim_{x \to \pi/4^+} \cos(x) = \sqrt{2}/2$$

$$f(\pi/4) = c = \frac{\sqrt{2}}{2}$$

$$f(x) = \begin{cases} \sin(x), & \text{if } x < \pi/4 \\ \cos(x), & \text{if } x > \pi/4 \end{cases}$$

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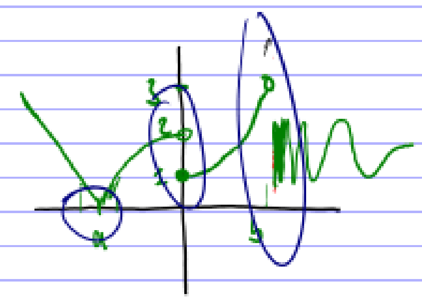
Panel 4

Name: ____________

Quiz #3

① Consider the graph below.



a) $\lim_{x \to a} f(x) = 0$

b) $\lim_{x \to b^-} f(x) = 3$

c) is f cont. at $x = a$? If not, what type: yes

d) is f cont. at $x = 0$? If not, what type: jump

e) is f cont. at $x = b$? If not, what type: essential

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Panel 5

② Where are the following functions continuous?

a) $f(x) = x^5 + 7x^2 - 9x + 3$ all $x \in \mathbb{R}$

$[0,1) \cup (1,\infty)$

b) $g(x) = \frac{\sqrt{x}}{x-1}$ $\{x \geq 0\} - \{1\}$

③ If $f(x) = \begin{cases} \frac{x^2-x}{x-1} & \text{if } x \neq 1 \\ c = 1 & \text{if } x = 1 \end{cases}$ choose a value for c so that f is cont. at $x=1$

$$\lim_{x\to 1} \frac{x^2-x}{x-1} = \lim_{x\to 1} \frac{x\cancel{(x-1)}}{\cancel{(x-1)}} = 1$$

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Panel 6

Find the solution for $x^2=2$ $(\sqrt{2}=x)$

$f(x)=x^2-2$ Want $f(x)=0$

$a=1 \Rightarrow f(1)=-1$

$b=2 \Rightarrow f(2)=+2$

$\Rightarrow$ There is c with $f(c)=0$, guess: 1.5

$f(1.5)=0.25 > 0$

$f(1)<0,\ f(1.5)>0$

$\Rightarrow c = 1.25$ $\Rightarrow f(1.25)<0$

$f(1.25)<0,\ f(1.5)>0$ $\Rightarrow c = \frac{1.25+1.5}{2} = 1.375$

$f(1.375)<0$

$f(1.375)<0,\ f(1.5)>0$

$c = \frac{1.375+1.5}{2} = 1.4375$

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Panel 7

Bisection Method: Solve $f(x)=0$, f cont.

① Find a, b with $f(a), f(b)$ different signs

② Compute $c =$ and find $f(c)$

if

else if

else if

if interested, let me know

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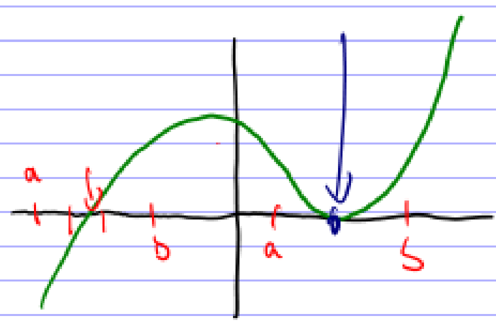
Panel 8

Bisection Method: easy to program

but requires $f(a), f(b)$ with different signs

does not find it!



more elaborate: Newton's method

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Panel 9

Limits at Infinity: a) value of $f(x) \to \infty$

b) value of $x \to \infty$

Ex.

$$\lim_{x \to 3^+} \frac{2x}{x-3} = +\infty$$

$$\lim_{x \to 3} \frac{2x}{x-3} = \text{d.n.e.}$$

$$\lim_{x \to 3^-} \frac{2x}{x-3} = -\infty$$

$$\lim_{x \to 3^+} \frac{2x}{(x-3)^2} = +\infty$$

$$\lim_{x \to 3} \frac{2x}{(x-3)^2} = \infty$$

$$\lim_{x \to 3^-} \frac{2x}{(x-3)^2} = +\infty$$

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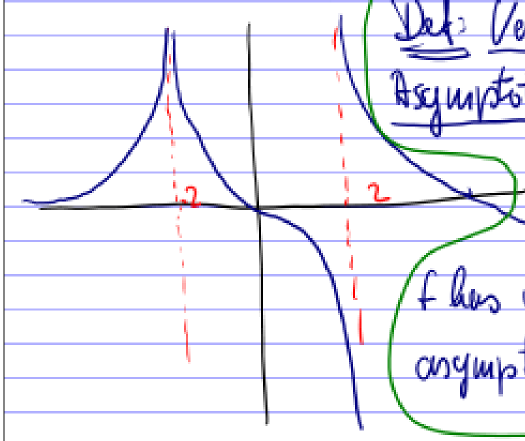
Panel 10



Def: Vertical Asymptote: If $\lim_{x \to c} f(x) = \pm\infty$ then f has vertical asymptote

$$\lim_{x \to -2^-} f(x) = +\infty$$

$$\lim_{x \to -2^+} f(x) = +\infty$$

$$\lim_{x \to 2^-} f(x) = -\infty$$

$$\lim_{x \to 2^+} f(x) = +\infty$$

$$\lim_{x \to 2} f(x) = \text{d.n.e.}$$

$$\lim_{x \to -2} f(x) = +\infty$$

$$\lim_{x \to 0} f(x) = 0$$

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Panel 11

Def: Let f be defined in $[a, \infty)$. Then

$$\lim_{x \to \infty} f(x) = L$$

means that $f(x)$ gets closer to L as x gets bigger, and bigger, and bigger, ...

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Panel 12

Algebraic limits at infinity

Think about these?

$$\lim_{x \to \infty} \frac{3x^2 - x - 2}{5x^2 + 4x + 1}$$

$$\lim_{x \to -\infty} \frac{3x^2 - 7}{5x^4 - 3x^2 + 9}$$

$$\lim_{x \to \infty} \frac{5x^3 - 7x^2 + 9}{4x + 7}$$

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