Panel 1

Math 1501: Last time

Limits $\lim_{x \to a} f(x) = L$

$\lim_{x \to a^-} f(x)$ (left-side) $\lim_{x \to a^+} f(x)$ (right-side)

To find limits:

① Plug in $x = a$ (don't tell) - if it works ✓

② $\frac{0}{\#} = 0$, $\frac{\#}{0}$ = d.n.e

③ $\frac{0}{0}$ = more work — factoring, rationalize, tricks

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Panel 2

$$f(x) = \begin{cases} 2-x & , x < -1 \\ x & , -1 \le x < 1 \\ (x-1)^2 & , x \ge 1 \end{cases}$$

$\lim_{x \to 2} f(x)$

$\lim_{x \to a} f(x)$

$\lim_{x \to -1} f(x) = ☹ \text{ d.n.e}$

$\lim_{x \to 0} f(x) = \lim_{x \to 0} x = 0$

$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} (2-x) = 3$

$\lim_{x \to -1^+} f(x) = \lim_{x \to 1^+} x = -1$

$\lim_{x \to 1} f(x) = ☹ \text{ d.n.e}$

$\lim_{x \to 1^-} f(x) = 1$ (x)

$\lim_{x \to 1^+} f(x) = 0$ ($(x-1)^2$)

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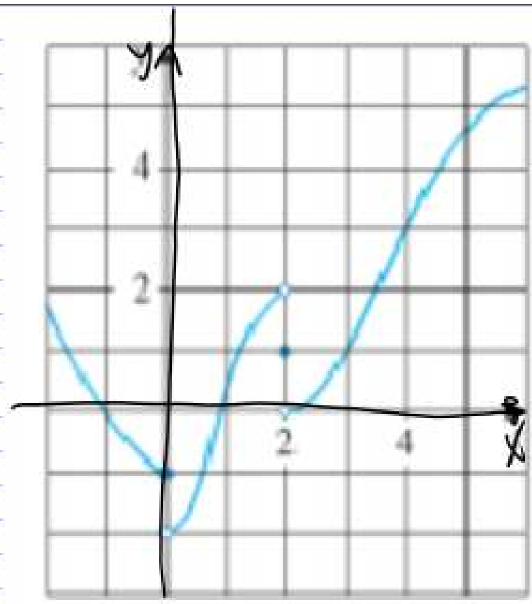
Panel 3



$\lim_{x\to -1^-} f(x) = 0$

$\lim_{x\to 0^+} f(x) = -2$

$\lim_{x\to 2^-} f(x) = 2$

$f(2) = 1$

$\lim_{x\to 4^+} f(x) = 3$

$\lim_{x\to 0} f(x) = \ddot{\smile}$ $\lim_{x\to 2} f(x) = \ddot{\frown}$ $\lim_{x\to 4} f(x) = 3$

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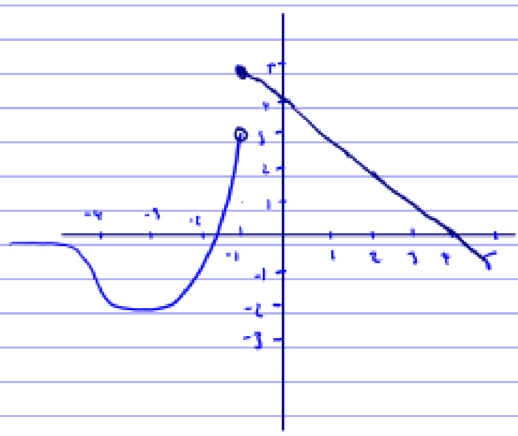
Panel 4

Quiz #2

Name: __________

① The graph of a function is shown below. Find:



a) $\lim_{x\to 4} f(x) =$

b) $\lim_{x\to -1^-} f(x) =$

c) $\lim_{x\to 0^+} f(x) =$

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Panel 5

② Compute the following limits:

a) $\lim_{x \to 2} \frac{x^2+1}{x+2} = \frac{5}{4}$

b) $\lim_{x \to 2} \frac{x^2-4}{x-2}$ $\quad \frac{(x-2)(x+2)}{(x-2)} \to 4$

c) $\lim_{x \to 9} \frac{\sqrt{x}-3}{x-9} \cdot \frac{(\sqrt{x}+3)}{(\sqrt{x}+3)} = \frac{\cancel{x-9}}{\cancel{(x-9)}(\sqrt{x}+3)} = 1/6$

$x^2 = 9 \quad \pm\sqrt{9} = x \qquad \sqrt{x} = 3$ for $x = 9$

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Panel 6

Piecewise defined functions (again)

$$f(x) = \begin{cases} \frac{x^2-x-6}{x-3} & \text{if } x \neq 3 \\ \cancel{10}\ 5 & \text{if } x = 3 \end{cases} \qquad g(x) = \begin{cases} x^2-5 & \text{if } x \geq 0 \\ x+1 & \text{if } x < 0 \end{cases}$$

a) $\lim_{x \to 0} f(x) = \frac{-6}{-3} = 2$

a) $\lim_{x \to 3} g(x) = 9-5 = 4$

b) $\lim_{x \to 3} f(x) =$

$\lim_{x \to 3} \frac{x^2-x-6}{x-3} = \lim_{x \to 3} \frac{\cancel{(x-3)}(x+2)}{\cancel{(x-3)}}$

$= 5$

b) $\lim_{x \to 0} g(x) = \text{d.n.e}$

$\lim_{x \to 0^+} x^2-5 = -5$

$\lim_{x \to 0^-} x+1 = 1$

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Panel 7

What would you pick for c if you had a choice?

$$f(x) = \begin{cases} \frac{x^3-1}{x-1} & \text{if } x \neq 1 \\ \cancel{x}\ 3 & \text{if } x = 1 \end{cases}$$

$$\lim_{x \to 1} f(x) = \lim_{x \to 1} \frac{x^3-1}{x-1} = \lim_{x \to 1} \frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{(x-1)}} = 3$$

Pick $c = 3$!

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Panel 8

Limits formally

$\lim_{x \to a} f(x) = L$: as x closes in on a, f(x) gets closer to L

No good definition!

Def. $\lim_{x \to a} f(x) = L$ means: given any $\varepsilon > 0$

you can find a $\delta > 0$ such that whenever

$|x-a| < \delta$ implies that $|f(x) - L| < \varepsilon$

Prove: $\lim_{x \to 1} 2x+1 = 3$

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Panel 9

Prove that $\lim_{x \to 1} 2x+1 = 3$

If for every number $\varepsilon > 0$ there is a $\delta > 0$
such that if $|x-a| < \delta$
then $|f(x)-L| < \varepsilon$

Take any # $\varepsilon > 0$ (e.g. 0.1), you have to find $\delta > 0$
s.t. if $|x-1| < \delta \Rightarrow |(2x+1)-3| < \varepsilon$ (0.1)
if $|x-1| < \delta \Rightarrow |2x-2| < \varepsilon$

$2|x-1| < \varepsilon$ (= 0.1)

pick $\delta = \varepsilon/2$!

$|x-1| < \frac{\varepsilon}{2}$ (= 0.01)

$\varepsilon = 0.005 \Rightarrow \delta = 0.0025$

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Panel 10

Prove that $\lim_{x \to 2} 4x+1 =$

leave to Math geeks

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Panel 11

Theorems about Limits

Suppose $\lim\limits_{x \to a} f(x) = M$ and $\lim\limits_{x \to a} g(x) = N$. Then:

$$\lim_{x \to a} f(x) + g(x) = M + N$$

$$\lim_{x \to a} f(x) - g(x) = M - N$$

$$\lim_{x \to a} f(x) \cdot g(x) = M \cdot N$$

$$\lim_{x \to a} f(x) / g(x) = M/N \quad \text{, only if } N \neq 0$$

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Panel 12

The Squeezing Theorem

If $g(x) \le f(x) \le h(x)$

and if $\lim\limits_{x \to a} g(x) = L = \lim\limits_{x \to a} h(x)$ then

$$\lim_{x \to a} f(x) = L$$

Ex: $\lim\limits_{x \to 0} x \sin\left(\frac{\pi}{x}\right) = \left(\lim\limits_{x \to 0} x\right)\left(\lim\limits_{x \to 0} \sin\left(\frac{\pi}{x}\right)\right) \cancel{= 0 \cdot \# = 0}$ done

$$-x \le x \sin\left(\frac{\pi}{x}\right) \le x \qquad \left(\left|\sin(\ )\right| \le 1\right)$$

$x \to 0$ 0 $x \to 0$

Thus $\lim\limits_{x \to 0} x \sin\left(\frac{\pi}{x}\right) = 0$

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Panel 13

$$\lim_{x\to 0} \frac{\sin(x)}{x} = 1 \qquad -\frac{1}{x} \le \frac{\sin(x)}{x} \le \frac{1}{x}$$

$-\frac{1}{x} \to -\infty$, $\frac{1}{x} \to \infty$

Squeeze thm doesn't work, but it is right

$u = 5x$

$$\lim_{x\to 0} \frac{\sin(5x)}{x} = \lim_{u\to 0} \frac{\sin(u)}{u/5} = \lim_{u\to 0} 5 \cdot \frac{\sin(u)}{u} = 5 \cdot 1 = 5$$

$$\lim_{x\to 0} \frac{\sin(x)}{5x} = \lim_{x\to 0} \frac{1}{5} \cdot \frac{\sin(x)}{x} = \frac{1}{5} \cdot 1$$

$$\lim_{x\to 0} \frac{\sin(5x)}{7x} = \frac{1}{7} \cdot \lim \frac{\sin(5x)}{x} = \frac{5}{7}$$

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Panel 14

$$\lim_{x\to 0} \frac{\sin(3x)}{\sin(2x)} = \lim \frac{\frac{\sin(3x)}{3x} \cdot 3\cancel{x}}{\frac{\sin(2x)}{2x} \cdot 2\cancel{x}} =$$

$$= \frac{\lim \frac{\sin(3x)}{3x}}{\lim \frac{\sin(2x)}{2x}} \cdot \frac{3}{2} = \frac{1 \cdot 3}{1 \cdot 2} = \frac{3}{2}$$

$$\lim_{x\to 0} \frac{|x|}{x} = ?$$

HW

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