Panel 1

Math 1501 - Last time

Quiz #1 - ✓

Shifting and Stretching ✓

Symmetry about y-axis: $f(-x) = f(x)$ even

about x-axis not function

about origin $f(-x) = -f(x)$ odd

Ready for Calculus!

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Panel 2

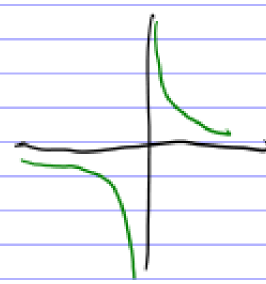
Most important concept in Calculus:

$f(x) = \frac{1}{x}$, $x \neq 0$.

What happens if x is close to zero?

x	$f(x)$
$\frac{1}{2} = 0.5$	2
$\frac{1}{10} = 0.1$	10
0.01	100
0.001	1000
	$\vdots$
	∞

x	$f(x)$
-0.1	-10
-0.01	-100
-0.001	-1000
	$\vdots$
	$-\infty$



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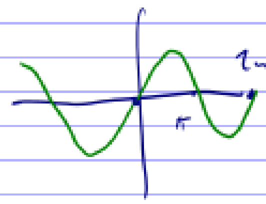
Panel 3

Let's do this again for $f(x) = \frac{\sin(x)}{x}$

which is also undefined for $x = 0$



x	$f(x)$
0.1	0.998..
0.01	0.99998..
0.001	0.9999998..

$\downarrow$

1

x	$f(x)$
−0.1	0.998
−0.01	0.99998
−0.001	0.9999998

$\downarrow$

1

$\sin(-x) = -\sin(x)$

$$\frac{\sin(-0.01)}{-0.01} = \frac{\cancel{-}\sin(0.01)}{\cancel{-}0.01} = \frac{\sin(0.01)}{0.01}$$

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Panel 4

Def: "The limit of $f(x)$ as x approaches a is L"
means: as x gets closer and closer to a, $f(x)$ gets closer and closer to L. We write

$$\lim_{x \to a} f(x) = L$$

Ex: $\lim_{x \to 0} \frac{\sin(x)}{x} = 1$

Ex: $\lim_{x \to 2} (x^2 - x + 2) = 4$

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Panel 5

Ex:

$$\lim_{x \to 1} \frac{x-1}{x^2-1} = \lim_{x \to 1} \frac{\cancel{x-1}}{(x+1)\cancel{(x-1)}} = \lim_{x \to 1} \frac{1}{x+1} = \frac{1}{2}$$

$$\lim_{x \to 0} \frac{\sqrt{x^2+9}-3}{x^2} = \frac{1}{6}$$

?

wolfram alpha

x	f(x)
0.1	
0.01	
0.001	0.166...

$$\lim_{x \to 1} \sin\left(\frac{\pi}{x}\right) = 0$$

no problem for $x = 1$!

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Panel 6

Why do we have to look for a pattern? Just use one really "close" number and get it over with!

$$\lim_{x \to 0} \sin\left(\frac{\pi}{x}\right) = \text{undefined}$$

	x	f(x)
$\frac{1}{10} =$	0.1	0
$\frac{1}{100} =$	0.01	0
$= \frac{1}{1000}$	0.001	0
		⋮
		0

x	f(x)
0.13	-0.92
0.027	-0.116
0.0089	0.904

no pattern

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Panel 7

Limits of Piecewise defined Functions:

$$H(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x > 0 \end{cases}$$

$H(1) = 1$, $H(-1) = 0$, $H(0) =$ undefined

$\lim_{x \to 0} H(x) =$ depends whether x is pos or neg!

$\Rightarrow$ One-sided limits

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Panel 8

One-Sided Limits

$\lim_{x \to a^+} f(x) = L$ means x close to a but $x > a$

$\lim_{x \to a^-} f(x) = L$ means x close to a but $x < a$

$\lim_{x \to 0^+} H(x) = 1$ and $\lim_{x \to 0^-} H(x) = 0$

$\lim_{x \to 0} H(x) =$ does not exist

Thm: $\lim_{x \to a} f(x) = L$ iff $\lim_{x \to a^+} f(x) = L$ and $\lim_{x \to a^-} f(x) = L$

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Panel 9

More Piecewise defined functions

$$f(x) = \begin{cases} \dfrac{x^2-x-6}{x-3} & \text{if } x \neq 3 \\ 15 & \text{if } x = 3 \end{cases}$$

a) $\lim_{x \to 0} f(x) = \frac{-6}{-3} = 2$

b) $\lim_{x \to 3} f(x) = \lim_{x \to 3} \frac{x^2-x-6}{x-3} = \lim_{x \to 3} \frac{\cancel{(x-3)}(x+2)}{\cancel{x-3}} = 5$

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Panel 10

$$f(x) = \begin{cases} x^2-1 & \text{if } x < 1 \\ 2x+3 & \text{if } x \geq 1 \end{cases}$$

$\lim_{x \to 1} f(x) =$ d.n.e. :(

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x^2-1 = 0$

$\neq$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} 2x+3 = 5$

$$g(x) = \begin{cases} \dfrac{\sin(x)}{x} & \text{if } x \neq 0 \\ 16 & \text{if } x = 0 \end{cases}$$

$\lim_{x \to 0} g(x) =$

$\lim_{x \to 0} \frac{\sin(x)}{x} = 1$

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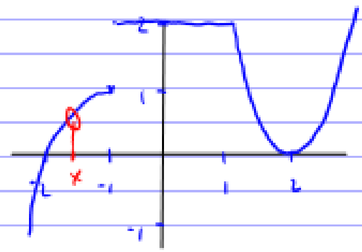
Panel 11

Limits Graphically:



$\lim_{x \to -2} f(x) = 0$

$\lim_{x \to -1} f(x) = \text{dne.}$

$\lim_{x \to -1^-} f(x) = 1 \qquad \lim_{x \to -1^+} f(x) = 2$

$\lim_{x \to -1^-} f(x) = 1$

$\lim_{x \to 0} f(x) = 2$

$\lim_{x \to 2} f(x) = 0$

$\lim_{x \to -1^+} f(x) = 2$

$\lim_{x \to 2^-} f(x) = 0$

$\lim_{x \to 2^+} f(x) = 0$

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Panel 12

Calculating Limits

Thm: Limit of sum / diff / product / ~~quotient~~ is the sum/diff / prod / ~~quotient~~ of the limits. For quotients, the denom. must be non-zero.

Find the following limits:

$\lim_{x \to 5} (2x^2 - 3x + 4) = 50 - 15 + 4$

$\lim_{x \to -2} \frac{x^3 + 2x^2 - 1}{5 - 3x} = \frac{-1}{11}$

$\lim_{t \to 0} \sqrt{t^2 + 9} = 3$

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Panel 13

Tricky vs. Simple Limits

$$\lim_{x\to 0} \frac{x^2-1}{x-1} = \frac{-1}{-1} = 1 \qquad \lim_{x\to 1} \frac{x^2-1}{x-1} = \lim_{x\to 1} \frac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}} = 2$$

$$\lim_{h\to 0} \frac{(3+h)^2-9}{h} = \lim_{h\to 0} \frac{\cancel{9}+6h+h^2-\cancel{9}}{h} = \lim_{h\to 0} \frac{\cancel{h}(6+h)}{\cancel{h}} = 6$$

$$\lim_{t\to 0} \frac{(\sqrt{t^2+9}-3)}{t^2}\frac{(\sqrt{t^2+9}+3)}{(\sqrt{t^2+9}+3)} = \lim_{t\to 0} \frac{1\,\cancel{t^2}+\cancel{9}-\cancel{9}}{\cancel{t^2}(\sqrt{t^2+9}+3)} = \frac{1}{6}$$

$$\lim_{x\to 0} \frac{(\cos(x)-1)}{x}\frac{(\cos(x)+1)}{(\cos(x)+1)} = \lim_{x\to 0} \frac{\cos^2(x)-1}{x(\cos(x)+1)} = \lim \frac{\sin^2(x)}{x(\cos(x)+1)}$$

$$= \lim_{x\to 0} \frac{\sin(x)}{x} \cdot \frac{\sin(x)}{\cos(x)+1} = 1 \cdot 0 = 0$$

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Panel 14

Not all limits exist.

$$\lim_{x\to 1} \frac{x^2-1}{x-2} = \frac{0}{-1} = 0 \checkmark$$

$$\lim_{x\to 1} \frac{x^2-2}{x-1} = \frac{-1}{0} = \text{undef}$$

$$\lim_{x\to 1} \frac{x^2-1}{x-1} = \frac{0}{0} \qquad \text{more work}$$

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