Panel 1

Last time:

- Maple assignment ✓
- Fundamental thm. of Calculus, part I

f cont. $\Rightarrow \int_a^b f(t)\,dt = F(x)\Big|_a^b = F(b) - F(a)$

Fundamental thm. of Calculus, part 2

f cont. $\Rightarrow \cancel{\frac{d}{dx}} \cancel{\int_a^x} f(t)\,dt = f(x)$

Integration by Substitution — indefinite, definite

Properties of the Integral

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Panel 2

Properties of the Integral

a) $\int_a^a f(x)\,dx = 0$, $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

b) If $f \geq 0 \Rightarrow \int_a^b f(x)\,dx \geq 0$ (area)

c) If f is pos. + neg. $\Rightarrow \int_a^b f(x)\,dx$ is "signed" net area

d) $\int c\,f(x) + d \cdot g(x)\,dx = c \int f(x)\,dx + d \int g(x)\,dx$

e) $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

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Panel 3

Substitution

$$\int \sin(x)\,(1-\sin^2(x))\,dx = \int \sin(x)\,\overbrace{\cos^2(x)}^{u^2}\,dx$$

$u = 1-\sin^2(x)$, $du = -2\sin(x)\cdot\cos(x)\,dx$

$u = \sin(x)$, $du = \cos(x)\,dx$

Now. $\Rightarrow u = \cos^2(x)$, $du = -2\cos(x)\sin(x)\,dx$

$u = \sin(x)$, $du = \cos(x)\,dx$

$u = \cos(x)$, $du = -\sin(x)\,dx$ $\Rightarrow -du = \sin(x)\,dx$

$$\int \sin(x)\cos^2(x)\,dx = -\int u^2\,du = -\frac{1}{3}u^3 + C = -\frac{1}{3}\cos^3(x) + C$$

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Panel 4

Substitution

$$\int_{-\pi}^{\pi} 2x\cdot\cos(x^2)\,dx = \int_{-\pi}^{\pi}\cos(x^2)\,2x\,dx$$

Quiz Wed

$u = x^2$, $du = 2x\,dx$ — if $x = -\pi \Rightarrow u = \pi^2$; if $x = \pi \Rightarrow u = \pi^2$

$$\int_{-\pi}^{\pi} 2x\cos(x^2)\,dx = \begin{cases} \displaystyle\int_{x=-\pi}^{x=\pi}\cos(u)\,du = \sin(u)\Big|_{x=-\pi}^{x=+\pi} = \sin(x^2)\Big|_{-\pi}^{\pi} = \sin(\pi^2) - \sin(\pi^2) \\ \displaystyle\int_{\pi^2}^{\pi^2}\cos(u)\,du = 0 \end{cases}$$

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Panel 5

Fund. Thm 2

Define $\boxed{F(x) = \int_0^x \sin(t^2)\,dt}$. Find $F''(x)$

There is no antiderivative for integrand, i.e. no way to define F without integration!

$$\underline{F'(x)} = \cancel{\frac{d}{dx}} \cancel{\int_0^x} \sin(t^2)\,dt = \sin(x^2)$$

$$F''(x) = 2x \cdot \cos(x^2)$$

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Panel 6

Fund. Thm. 2

If $G(x) = \int_1^{x^3} \cos(t)\,dt$, find $G'(x)$

$$G(x) = \int_1^{x^3} \cos(t)\,dt = \sin(t)\Big|_{t=1}^{x^3} = \underline{\sin(x^3) - \sin(1)}$$

$$G'(x) = \cos(x^3) \cdot 3x^2$$

Theorem: If $G(x) = \int_a^{u(x)} f(t)\,dt$ then

$$G'(x) = f(u(x)) \cdot u'(x)$$

(extended Fund. Thm. of Calculus)

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Panel 7

Fund. Thm. 2

If $F(x) = \int_1^{\sin(x)} \tan(t)\,dt$, find $F'(x)$

$$\Rightarrow F'(x) = \cancel{\frac{d}{dx}} \cancel{\int_1^{\sin(t)}} \tan(t)\,dt = \tan(\sin(x)) \cdot \cos(x)$$

If $F(x) = \int_{x^2}^{x^3} \sec^2(t)\,dt$, find $F'(x)$

$$\Rightarrow \int_{x^2}^{x^3} \sec^2(t)\,dt = \int_{x^2}^{0} \sec^2(t)\,dt + \int_{0}^{x^3} \sec^2(t)\,dt =$$

$$= -\underline{\int_0^{x^2} \sec^2(t)\,dt} + \underline{\int_0^{x^3} \sec^2(t)\,dt} =$$

$$\Rightarrow F'(x) = -\sec^2(x^2)\cdot 2x + \sec^2(x^3)\cdot 3x^2$$

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Panel 8

$$\cancel{\frac{d}{dx}} \cancel{\int_5^{x}} \sin(t)\cos(t)\,dt = \sin(x)\cos(x)$$

$$\cancel{\frac{d}{dx}} \cancel{\int_5^{x^3}} \sin(t)\cos(t)\,dt = \sin(x^3)\cos(x^3)\cdot 3x^2$$

$$\frac{d}{dx} \int_x^{x^4} \sin(t)\cos(t)\,dt = \sin(x^4)\cos(x^4)\cdot 4x^3 - \sin(x)\cos(x)\cdot 1$$

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Panel 9

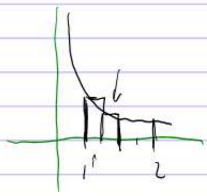
Def: Let $B(x) = \int_1^x \frac{1}{t}\,dt$, $x > 0$

Find, if possible, $B(1)$ ✓ and $B(2)$

Note: $\int \frac{1}{t}\,dt$ has no antiderivative!

$\int x^n\,dx = \frac{1}{n+1}x^{n+1}$ works for $n \neq -1$

$B(1) = \int_1^1 \frac{1}{t}\,dt = 0$, $B(2) = \int_1^2 \frac{1}{t}\,dt = (\text{approx}) \approx 0.69$



$\int_1^2 \frac{1}{t}\,dt \approx 0.25\cdot\big(f(1) + f(1.25) + f(1.5) + f(1.75)\big) =$

$0.25\left(\frac{1}{1} + \frac{1}{1.25} + \frac{1}{1.5} + \frac{1}{1.75}\right) \approx \underline{0.69}$

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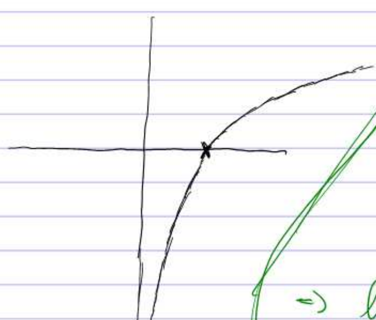
Panel 10

Ex: Define $B(x) = \int_1^x \frac{1}{t}\,dt$, $x > 0$. Sketch $B(x)$

Know. $B(1) = 0$, $B'(x) = \frac{1}{x}$, $B''(x) = -\frac{1}{x^2}$, $x > 0$

$\Rightarrow B'(x) > 0$, $B''(x) < 0$

$\Rightarrow B$ increasing, concave down



Definition:

$\ln(x) = \int_1^x \frac{1}{t}\,dt$ is def. of natural logarithm.

$\Rightarrow \ln(1) = 0$, $\frac{d}{dx}\ln(x) = \frac{1}{x}$

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Panel 11

Prove: f even $\Rightarrow \int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$ extra credit ?!!

?

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Panel 12

Mean Value Theorem for Integration

Reviews Intermediate Value thm:

$f(a) < 0,\ f(b) > 0 \Rightarrow f(c) = 0$ for c

Rolle's theorem:

$f(a) = f(b), \Rightarrow f'(c) = 0$ for c

Mean Value theorem:

$\frac{f(b)-f(a)}{b-a} = \frac{1}{b-a}\cdot(f(b)-f(a)) = f'(c)$ for c

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Panel 13

Mean Value Theorem for Integration:

f cont. on $[a,b]$. Then there is a $c \in [a,b]$

with $\frac{1}{b-a} \int_a^b f(x)\,dx = f(c)$

where $\frac{1}{b-a} \int_a^b f(x)\,dx$ is called *average* of a function f on $[a,b]$.

Nice geometric interpretation!

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Panel 14

Mean Value Thm: $\frac{1}{b-a} \int_a^b f(x)\,dx = f(c)$ for some c

Ex: Find c for average value of $f(x) = 1+x^2$ over $[-1,2]$.

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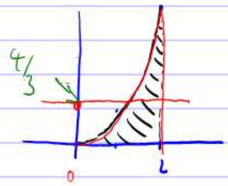
Panel 15

Balance $f(x) = x^2$ between $[0,2]$ ($a = 0$, $b = 2$)



Geometric interpretation of MVT for Int.

$\frac{1}{b-a}\int_a^b f(x)\,dx$ balances a function

$$\frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{2-0}\int_0^2 x^2\,dx = \frac{1}{2}\cdot\frac{1}{3}x^3\Big|_0^2 = \frac{8}{6} = \frac{4}{3}$$

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Panel 16

Topics

1. Limits + Continuity ⇒ Exam 1
2. Differentiation ⇒ Exam 2
3. Integration ⇒ Exam 3

on

Quiz on Wed

Mon Dec 3

Wed Dec 5

Final: Dec 18, 10:45 (Tuesday)

cummulative

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Panel 17

Inverse Functions (Chapter 5)

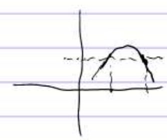
Want to explore new functions from existing ones, but not all functions are suitable:

Method 1: use integration as in $\ln(x) = \int_1^x \frac{1}{t}\,dt$

Method 2: use inverse functions

Def: A function f is called one-to-one (1-1) if it never takes on the same value twice, i.e.

if $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$



Ex: $f(x) = x^2$ $\quad f(2) = f(-2)$, but $2 \neq -2 \Rightarrow$

f is NOT one-to-one

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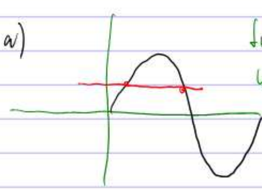
Panel 18

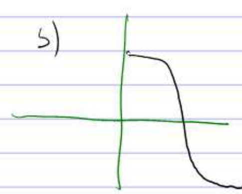
Vertical Line Test: If every vertical line intersects a graph at most once, the graph represents a function.

Horizontal Line Test: If every horizontal line intersects a graph at most once, the graph is a 1-1 function

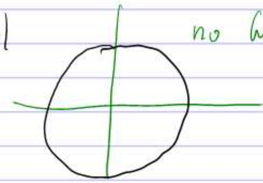
a)



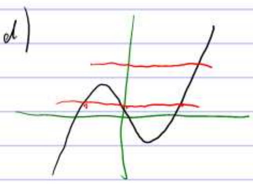
b)



c)



d)



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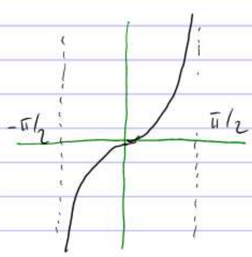
Panel 19

Theorem: If f is diff'ble and $f'(x) > 0$ (or $f'(x) < 0$) then f is 1-1.

If $f'(x) > 0 \Rightarrow f$ is increasing

Ex: $f(x) = \tan(x) = \frac{\sin(x)}{\cos(x)}$, $x \in (-\frac{\pi}{2}, \frac{\pi}{2})$ is 1-1

because $f'(x) = \sec^2(x) > 0$



$\int \tan(x)\,dx = \int \frac{\sin(x)}{\cos(x)} \cdot dx = -\int \frac{1}{u}\, du =$

$u = \cos(x)$ $= -\ln|u| + C =$

$du = -\sin(x)\,dx$ $= -\ln(\cos(x)) + C$

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Panel 20

Def:

See you Wed – Quiz

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