Panel 1

Last time - nothing new!

New terminology:

$\int f(x)dx$ = indefinite integral = antiderivative

Fundamental Theorem of Calculus = Evaluation thm

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$$

also: $\int_a^b f(x)dx$ represents area under curve if f is positive ($f>0$)

1

Panel 2

Laptops next:

start Maple!

Plot x^2 and $1/x$ in one coordinate system

plot({x^2, 1/x}, x=-5..5, y=-8..8);

$\int_0^5 x\sin(x)\,dx$ = int(x sin(x), x=0..5)

2

Panel 3

Integration by Substitution

"What if" - integration trick

Ex: $\int 2x\sqrt{1+x^2}\,dx = \int \sqrt{u}\,du = \frac{2}{3}u^{3/2}+C$

$1+x^2 = u$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$= \frac{2}{3}(1+x^2)^{3/2}+C$

3

Panel 4

$\int x^3\cos(x^4+2)\,dx = \int x^3 \cancel{\cos(u)}\,dx$ too soon

Let $u = x^4+2$

$\frac{du}{dx} = 4x^3$

$du = 4x^3\,dx$

$\frac{1}{4}du = x^3\,dx$

$= \int \frac{1}{4}\cos(u)\,du =$

$= \frac{1}{4}\sin(u)+C$

$= \frac{1}{4}\sin(x^4+2)+C$

4

Panel 5

$$\int x^3 \cos(x^4+2)\,dx = \int u \cos(x^4+2)\,dx \quad ?$$

$u = x^3$

$\frac{du}{dx} = 3x^2$

$du = 3x^2\,dx$

dead-end street

5

Panel 6

$$\int \sqrt{2x+1}\,dx = \int \frac{1}{2}\sqrt{u}\,du \quad \checkmark$$

$u = 2x+1$

$\frac{du}{dx} = 2$

$du = 2\,dx$

$\frac{1}{2}du = dx$

$$\int \frac{x}{\sqrt{1-4x^2}}\,dx$$

$u = x \rightarrow \frac{du}{dx} = 1$

$\Rightarrow u = 1-4x^2 \rightarrow \frac{du}{dx} = -8x$

$u = \sqrt{1-4x^2} \rightarrow \frac{du}{dx} = \times$

$u = \frac{x}{\sqrt{1-4x^2}} \rightarrow \frac{du}{dx} = \times$

6