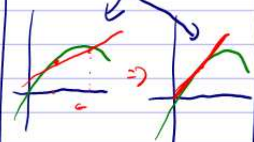
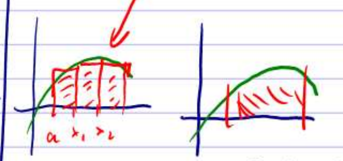
Panel 1

Last time: the Integral

	cont	deriv.	integration
def:	$\lim\limits_{x \to c} f(x) = f(c)$	$\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$	$\lim\limits_{n \to \infty} f(x_1)\Delta x_1 + \dots + f(x_n)\Delta x_n$
geometry	 not cont.	 slope of tangent	 area under f (>0)
do it	X	Quotient, Prod. chain.	Evaluation theorem

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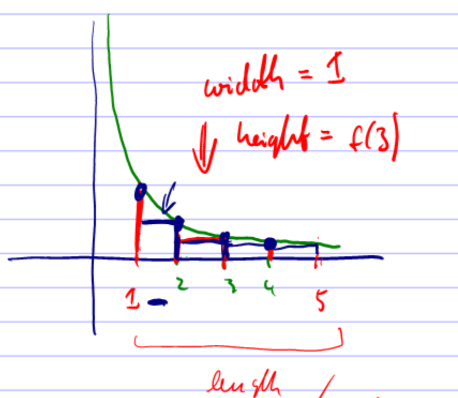
Panel 2

#3) Estimate area under $f(x) = 1/x$ from $x=1$ to $x=5$ using 4 rectangles and a) right endpoints, b) left ones.



area is approx:

a) $1 f(2) + 1 f(3) + 1 f(4) + 1 f(5) =$
$= 1/2 + 1/3 + 1/4 + 1/5 = A$

b) $1 \cdot f(1) + 1 f(2) + 1 \cdot f(3) + 1 \cdot f(4) =$
$= 1 + 1/2 + 1/3 + 1/4 = B$

$$A \le \int_1^5 \frac{1}{x}\,dx \le B$$

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Panel 3

Evaluation Theorem: $\int_a^b f(x)\,dx = F(b) - F(a)$ antiderivative

$$\int_1^5 \frac{1}{x}\,dx = ?$$

$$\int_1^3 \frac{1}{x^2}\,dx = -(3)^{-1} + (1)^{-1} = 1 - 1/3 = \underline{\underline{2/3}}$$

$$F(x) = -x^{-1} \xrightarrow{\text{deriv.}} -1(-1)x^{-2} = x^{-2} = 1/x^2 \checkmark$$

#17 canceled → bring your laptop, Maple

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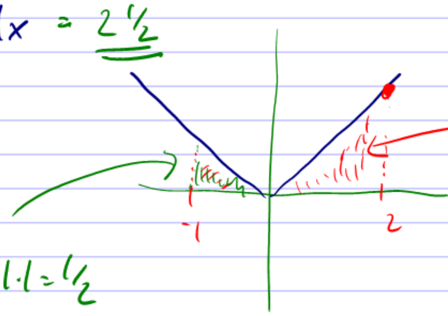
Panel 4

$$\int_{-1}^{2} |x|\,dx = \underline{\underline{2\tfrac{1}{2}}}$$



Area $\frac{1}{2}\, b \cdot h = 2$

$A = \frac{1}{2}\, 1 \cdot 1 = 1/2$

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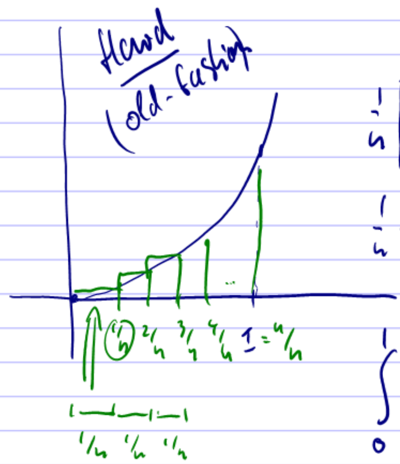
Panel 5

Area under x^3 from 0 to 1:

Answer is $\int_0^1 x^3\,dx = \frac{1}{4}$ (modern way)

Use definition to verify this answer:



$$f\left(\tfrac{1}{n}\right)\cdot\tfrac{1}{n} + f\left(\tfrac{2}{n}\right)\tfrac{1}{n} + f\left(\tfrac{3}{n}\right)\tfrac{1}{n} + \dots + f\left(\tfrac{n}{n}\right)\cdot\tfrac{1}{n} =$$

$$\frac{1}{n}\left[\left(\frac{1}{n}\right)^3 + \left(\frac{2}{n}\right)^3 + \left(\frac{3}{n}\right)^3 + \dots + \left(\frac{n}{n}\right)^3\right] =$$

$$\frac{1}{n}\cdot\frac{1}{n^3}\left(1^3+2^3+3^3+\dots+n^3\right) \overset{\text{book}}{=} \frac{1}{n^4}\left(\frac{n(n+1)}{2}\right)^2$$

$$\int_0^1 x^3\,dx = \lim_{n\to\infty}\frac{1}{n^4}\left(\frac{n(n+1)}{2}\right)^2 = \frac{1}{n^4}\frac{(n^2+n)^2}{4} = \frac{1}{4}$$

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Panel 6

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Panel 7

New terminology:

Fundamental Theorem of Calculus, part 1

If f is a function with anti-derivative F then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

We sometimes write

$$\int_a^b f(x)\,dx = F(x)\Big|_{x=a}^{x=b} = F(b) - F(a)$$

Ex: $\int_0^3 (x^3 - 6x)\,dx = \frac{1}{4}x^4 - 3x^2 \Big|_0^3 = \left(\frac{1}{4}\cdot 3^4 - 3\cdot 3^2\right) - 0 =$

$= \frac{1}{4}3^4 - 27$

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Panel 8

From now on we denote the antiderivative of f as

$$\int f(x)\,dx = F(x)$$

and call it "indefinite integral"

Ex: $\int \cos(x)\,dx = \sin(x) + C$ (antideriv.)

$$\int_\pi^{2\pi} \cos(x)\,dx = \sin(x)\Big|_\pi^{2\pi} = \sin(2\pi) - \sin(\pi) = 0$$

Note: A definite integral is a number!

An indef. integral is a function!

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Panel 9

Moments of Inertia:

$M_x = \int x f(x) dx$

next time → with Maple

Mathematica = Maple

Quiz on Wed, Laptops, check HW online!

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