Panel 1

Last time:

Review of local extrema

abs. extrema

incr / decr

concave up/down

asymptotes

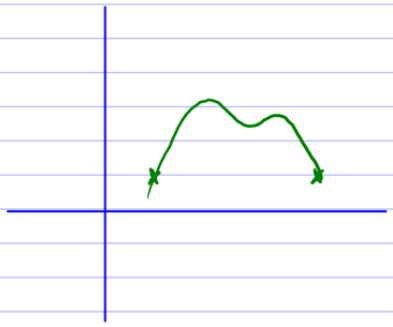
graphing

Various theorems

1

Panel 2

Rolle's Theorem: If f is differentiable on (a,b) and continuous on $[a,b]$, and then



Related to Rolle's thm is →

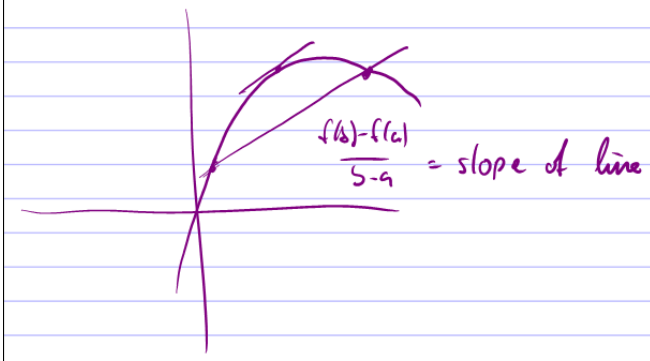
2

Panel 3

Mean Value Theorem: (MVT)

f diffbl on (a,b), cont on $[a,b]$. Then

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$



Ex: $f(x) = x^3 - x$, $a=0, b=2$ $\Rightarrow$ $c = \pm \frac{2}{\sqrt{3}}$

3

Panel 4

Proof of MVT:

Define $h(x) = f(x) - \left[f(a) + \frac{f(b)-f(a)}{b-a}(x-a)\right]$

$\Rightarrow$ $h(a) = 0 = h(b)$ $\Rightarrow$

$h'(c) = 0$ $\Rightarrow$

Result

#

Thm: If $f'(x) = 0$ $\forall x$ on (a,b) then f is constant.

take any x, y $\Rightarrow$ $f'(c) = \frac{f(b)-f(y)}{b-y} = 0$ $\Rightarrow$ $f(b) = f(y)$

no matter what x, y is!

4

Panel 5

Optimization Problems

A farmer has 2400 ft of fencing to enclose a rectangular plot bordering a river. No fence is needed on river. What dimensions will give the largest area?

5

Panel 6

A cylindrical can should hold 1 l of oil. Minimize cost of metal used to manufacture the can.

$V = \pi r^2 \cdot h = 1 \quad \Rightarrow h = \frac{1}{\pi r^2}$

$A = 2\pi r h + (\pi r^2) \cdot 2$

$= 2\pi \cdot \left(\frac{1}{\pi r^2}\right) + 2\pi r^2$

$= \frac{2}{r} + 2\pi r^2$

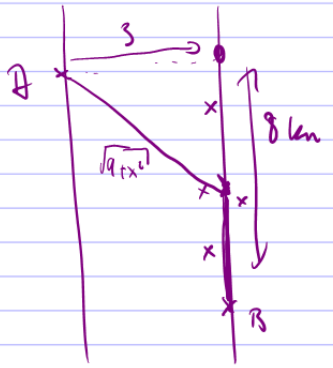
$A' = -\frac{2}{r^2} + 4\pi r = \frac{4\pi r^3 - 2}{r^2} = 0 \Rightarrow r^3 = \frac{1}{2\pi} \Rightarrow r = \sqrt[3]{\frac{1}{2\pi}}$

Why is it a minimum?

6

Panel 7

A man launches a boat from point A on a river bank, and wants to reach point B, 8 km downstream on opposite side. The river is 3 km wide. He can row 6 km/h and run 8 km/h. Where should he land to reach B fastest.



running dist: $8-x$

rowing dist: $\sqrt{9+x^2}$

speed = $\frac{\text{dist}}{\text{time}}$ $\Rightarrow$ time = $\frac{\text{dist}}{\text{speed}}$

$t_1 = \frac{1}{8}(8-x) + \frac{1}{6}\sqrt{9+x^2}$

$T'(x) = 0 \Leftrightarrow x = \frac{9}{\sqrt{7}}$

7

Panel 8

A store is selling 200 DVD burners a week at $350 each. Market research shows that each $10 rebate will sell 20 additional units per week. How large should rebate be to maximize the revenue.

x = # units sold per week, $p(x)$ price per unit

$\Rightarrow R(x) = x \cdot p(x)$ $p(x)$ was 350.

extra cost is $10 per 20 units over 200 or $\frac{10}{20}$ per unit over 200 or $\frac{10}{20}(x-200)$, $x \geq 200$

$\Rightarrow p(x) = 350 - \frac{10}{20}(x-200) = 450 - \frac{1}{2}x$

$\Rightarrow R(x) = x(450 - \frac{1}{2}x) = 450x - \frac{1}{2}x^2$

$R'(x) = 450 - x = 0 \rightarrow \boxed{x = 450} \Rightarrow p(450) = 225$

8

Panel 9

Antiderivatives

A function F is called antiderivative of f if

$F' = f$

Ex: $f(x) = x^2 \Rightarrow F(x) = \frac{1}{3}x^3$ also $G(x) = \frac{1}{3}x^3 + 1$

Any others?

Ex: Find F for $\sin(x)$ $\cos(x)$ x^{10} x^{-3} $\sqrt{x}$ x^4

$f'' = 12x^2 + 6x - 4$ find f!

9