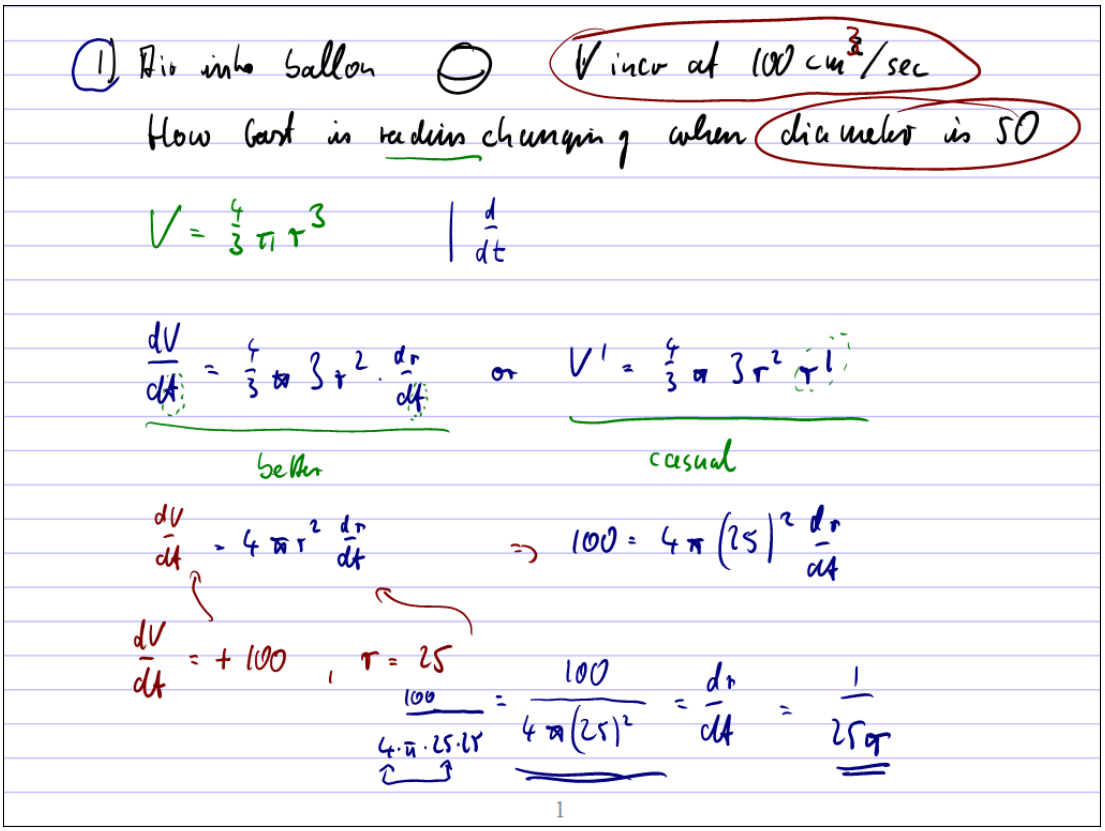
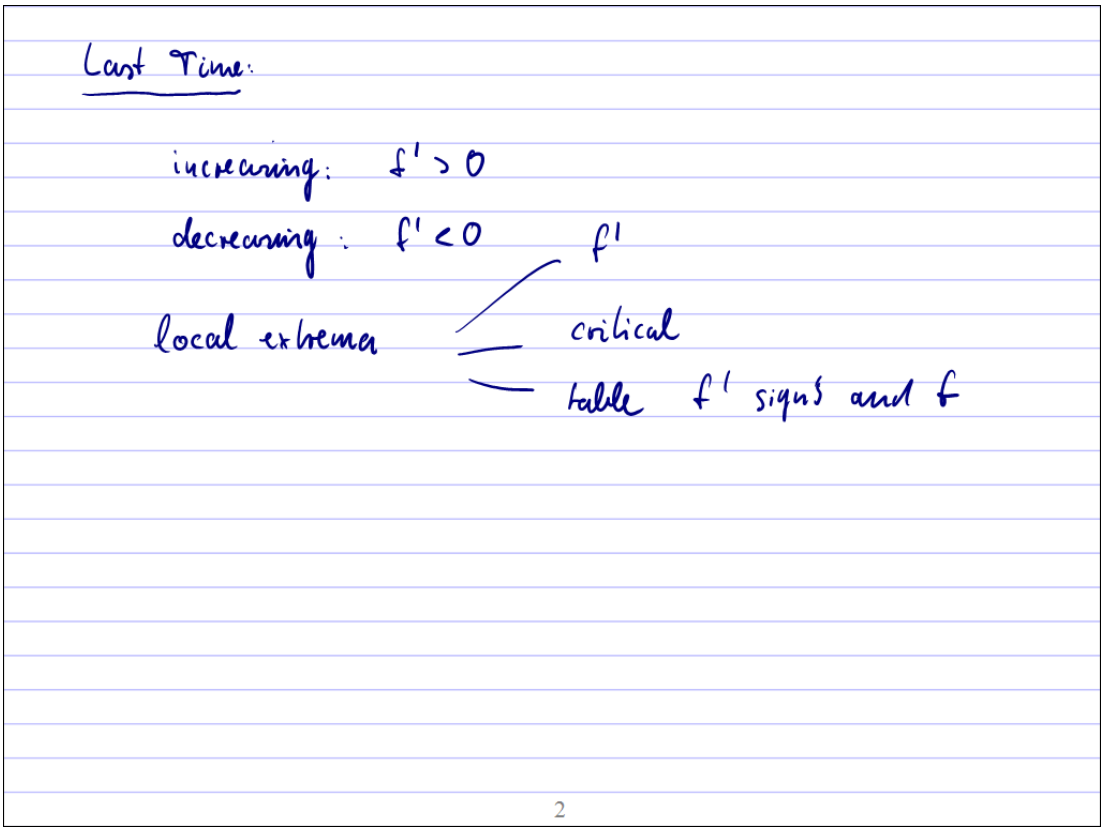
Panel 1



Panel 2



Panel 3

29) $g(y) = \dfrac{y-1}{y^2-y+1}$ find critical numbers

$g(x) = \dfrac{x-1}{x^2-x+1}$

$g'(x) = \dfrac{x^2-x+1-(x-1)(2x-1)}{(x^2-x+1)^2} = \dfrac{x^2-x+1-2x^2+x+2x-1}{(x^2-x+1)^2}$

$= \dfrac{-x^2+2x}{(x^2-x+1)^2} = 0 \rightarrow 2x-x^2=0$

$x(2-x)=0 \Rightarrow x=0,2$

f' d.ne. if $x^2-x+1=0$

$x_{1/2} =$

3

Panel 4

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

x^2-x+1

$x = \dfrac{1 \pm \sqrt{1-4}}{2} = \dfrac{1 \pm \sqrt{-3}}{2}$ not real

only crit. $x=0,2$

4

Panel 5

$f(x) = \frac{\sqrt[3]{x}}{1+x^2}$ find local extrema!

Q3 on Wed

$f'(x) = \frac{\frac{1}{3}x^{-2/3}(1+x^2) - x^{1/3}\cdot 2x}{(1+x^2)^2} \cdot \frac{3x^{2/3}}{3x^{2/3}} =$

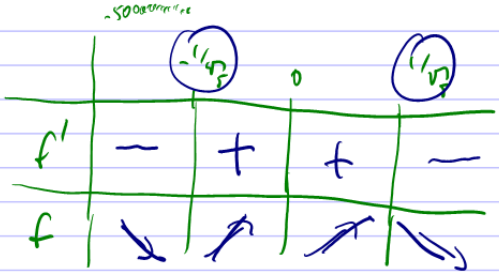
$= \frac{1+x^2-6x^2}{3x^{2/3}(1+x^2)^2} = \frac{1-5x^2}{3x^{2/3}(1+x^2)^2}$

critical

$x = 0$

$x = 1/\sqrt{5}$

$x = -1/\sqrt{5}$



$x = 0$ is nada

$x = 1/\sqrt{5}$ is local max

$x = -1/\sqrt{5}$ is local min

5

Panel 6

To find abs. extrema on $[a,b]$: If f is continuous on $[a,b]$ there is abs. max and min. (by theorem)

Ex: $f(x) = x^3 - 3x^2 + 1$ on $[-1,4]$

$f'(x) = 3x^2 - 6x$

$f'(x) = 3x(x-2) = 0$, $x = 0, 2$

x	$f(x)$	
0	1	
2	$8 - 12 + 1 = -3$	abs. min
−1	$-1 - 3 + 1 = -3$	
4	$64 - 48 + 1 = 15$	max abs.

1.) Find f'

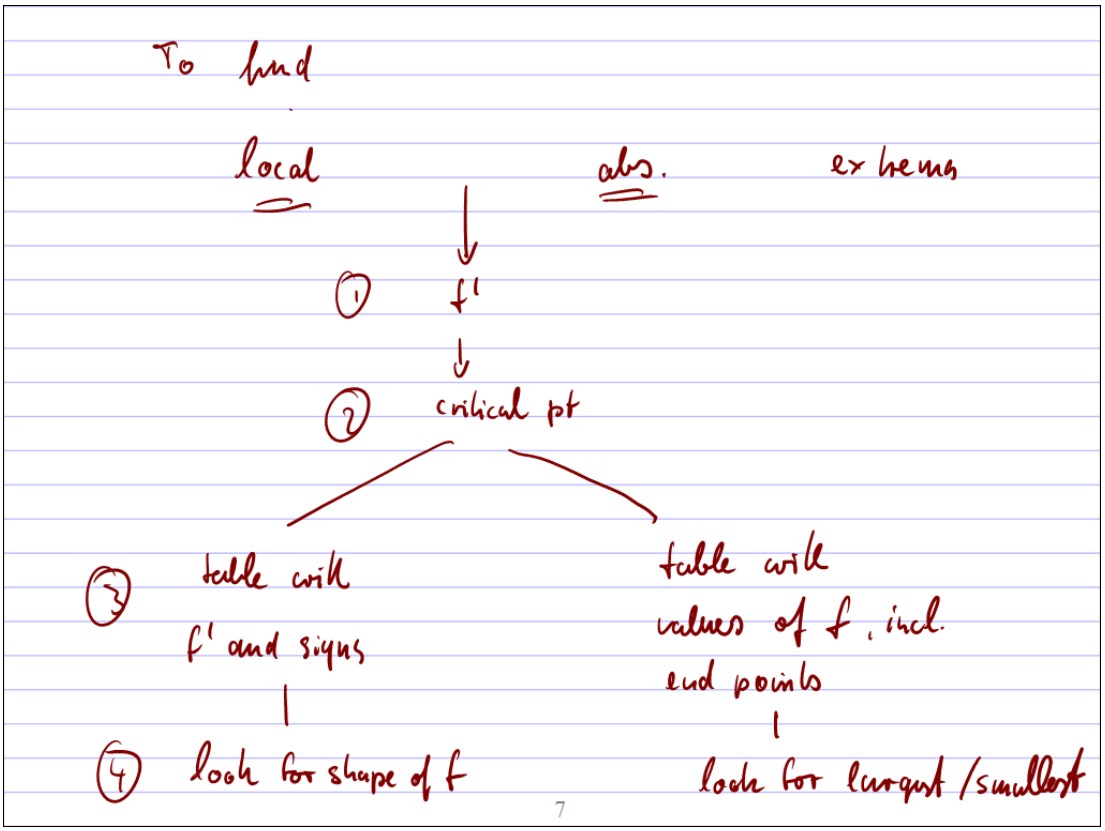
2.) Find critical ($f' = 0$, f' d.n.e.)

3.) Make table of values, incl. endpoints of interval

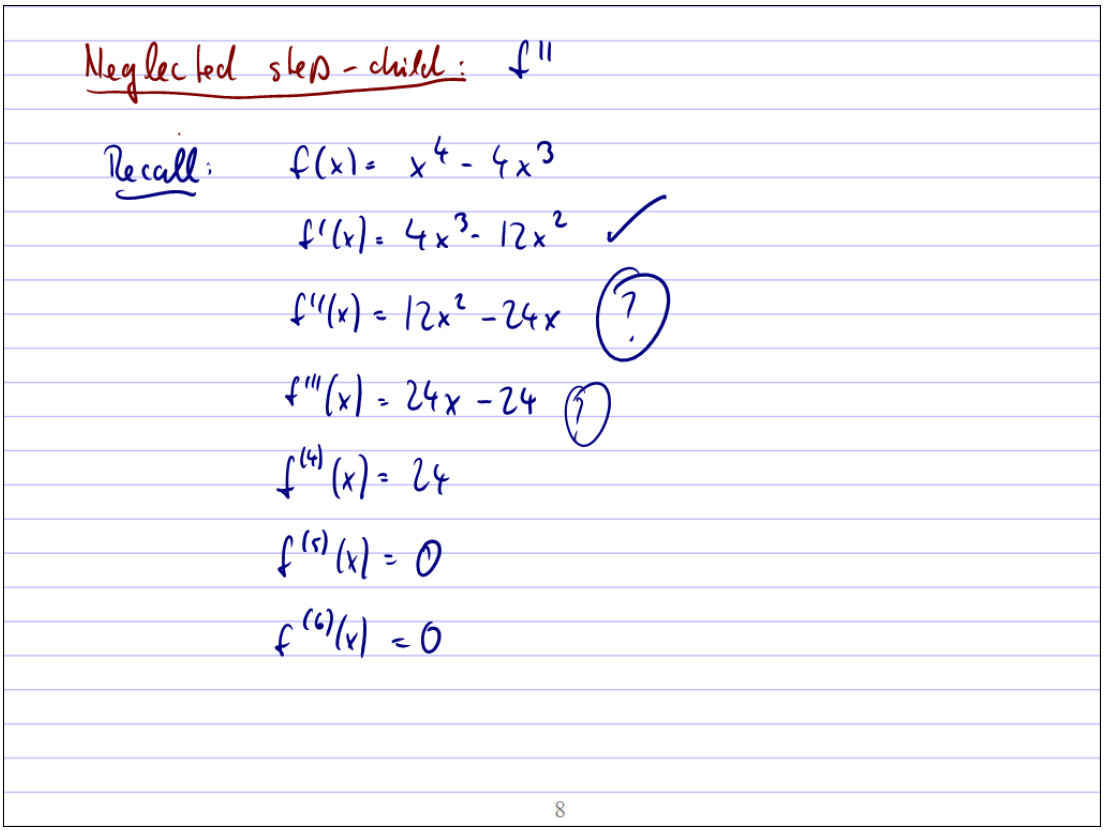
4) Pick largest/smallest

6

Panel 7



Panel 8

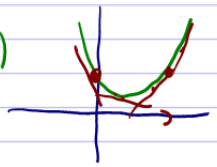


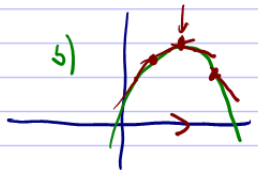
Panel 9

What is the significance of f''

if $f'' > 0 \Rightarrow f'$ increasing $\Rightarrow$ slope of f is incr

$f'' < 0 \Rightarrow f'$ decreasing $\Rightarrow$ slope of f is decr.

a)  concave up

b)  concave down

For which function is the slope increasing? for (a)

9

Panel 10

Def: If f is concave up: ∪

If f is concave down: ∩

Theorem: If $f'' > 0$ then f is concave up

If $f'' < 0$ then f is concave down

Points where concavity changes are called inflection points!

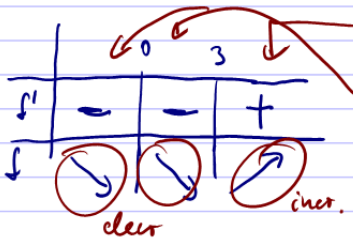
10

Panel 11

Ex: $f(x) = x^4 - 4x^3$, $f'(x) = 4x^3 - 12x^2 = 4x^2(x-3)$

a) find where f is incr. / decreasing

critical:
$x = 0$
$x = 3$

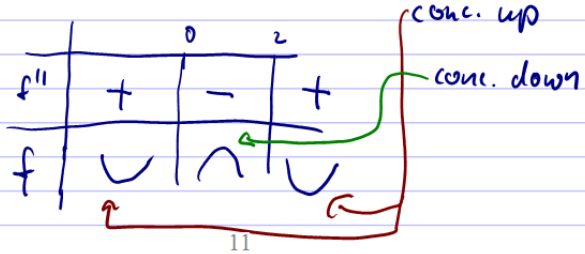


incr.: $(3, \infty)$
decr. $(-\infty, 0) \cup (0, 3)$

b) where is f concave up / down, respectively

$f''(x) = 12x^2 - 24x = 12x(x-2)$

poss. inflect.
$x = 0$
$x = 2$



11

Panel 12

local extrema: f', critical, table with signs of f' $\Rightarrow$ #

abs. extrema: f', critical, table of values + endp $\Rightarrow$ #

increasing / decreasing: f', critical, table $\Rightarrow$ interval

concavity: f', f'', poss. infl., table of f'' $\Rightarrow$ intervals

inflection points: f', f'', table of f'' $\Rightarrow$ numbers

Sketch a function

12

Panel 13

Ex. Find the graph of $f(x) = x^{2/3}(6-x)^{1/3}$

Hint: $f'(x) = \frac{4-x}{x^{1/3}(6-x)^{2/3}}$, $f''(x) = \frac{-8}{x^{4/3}(6-x)^{5/3}}$

critical: $x = 4$, $x = 0$, $x = 6$

poss. infl. $x = 0$, $x = 6$

	0	4	6
f'	−	+	−	−
f''	−	−	−	+
f	↘	↗	↘	∪

more discussion!

$x = 4$ local max, $x = 6$ inflection

13

Panel 14

When graphing function, also consider asymptotes

0.) Find asympt. if appropriate
   a) horiz: $\lim_{x \to \pm\infty} f(x) = L$, $y = L$
   b) vertical $f(x)$ undef.
1.) Find f' and simplify
   Find critical
2.) Find f'' and simplify
   Find poss. infl.
3.) Draw table with f', f'' signs and shape of f
4.) Draw the function

14

Panel 15

$f(x) = \frac{2x^2}{x^2-1}$, $f'(x) = \frac{-4x}{(x^2-1)^2}$, $f''(x) = \frac{12x^2+4}{(x^2-1)^3}$

Asymptotes: $\lim_{x\to\infty} \frac{2x^2}{x^2-1} = 2$, $y=2$ horiz. asympt.

$x=+1$, $x=-1$ are vert. asympt.

critical: $x=0$, $x=1$, $x=-1$

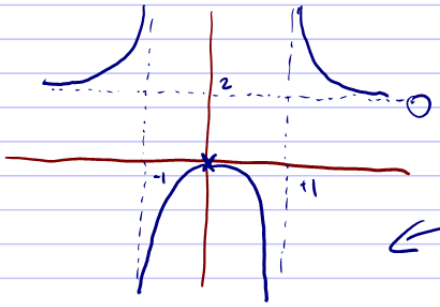
possible infl: $x=+1,-1$

	-1		0		1	
f'	+	+		−		−
f''	+		−		−	+
f	↑		↑		↓	↓

$x=0$ is max

$f(0)=0$

easy



hidey!

15

Panel 16

HW is posted

Q8 on Wed

Ex 2: Mon after Halloween

16