

Panel 1

Next Quiz

Related Rates ✓

linear equation of tangent

✓ Linear Approx  $f(x) \approx \underbrace{f'(a)(x-a)} + \underbrace{f(a)}$  for  $x$   
close to  $a$

Error Estimates

p

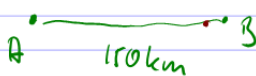
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Panel 2

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Ship A is 150 km west of Ship B  
at noon. A sails east at 35 km/h  
B sails north at 25 km/h. <sup>Rule</sup> Distance  
changing at 4 pm

noon



$$d^2 = 150^2 + 100^2 \quad \checkmark$$

$$d^2 = x^2 + y^2$$

$$2d(d') = 2x(x') + 2y(y')$$

$$\begin{array}{l} x' = 35 \\ y' = 25 \end{array} \quad \begin{array}{l} x = 10 \\ y = 100 \end{array} \quad \left| \text{Friday} \right.$$

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Panel 3

$f(x) \approx \underbrace{f'(a)}_{\downarrow} (x-a) + \underbrace{f(a)}_{\downarrow}$

$(2.001)^2$

$f(x) = x^2, f'(x) = 2x$   
 $a = 2$

$\Rightarrow x^2 \approx 4 \cdot (x-2) + 4$   
 $(2.001)^2 \approx 4 \cdot (2.001-2) + 4 =$   
 $= 4 \cdot 0.001 + 4 = \underline{4.004}$

$\Downarrow$

$\sqrt{99.81} \pm \sqrt{100 - 0.2}$

$f(x) = \sqrt{x}, f'(x) = \frac{1}{2\sqrt{x}}$   
 $a = 100$

$\sqrt{x} \approx \frac{1}{2\sqrt{100}} \cdot (x-100) + \sqrt{100} =$   
 $= \frac{1}{20} (x-100) + 10$

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Panel 4

square

How many square inches does my screen have?

measure  $l = 10$  with error  $0.2$  cm

What is error in area?

$A = l^2, \frac{dA}{dl} = 2l \Rightarrow dA = 2 \cdot l \cdot dl$

$\Rightarrow dA = 2 \cdot l \cdot dl = 2 \cdot 10 \cdot 0.2 = 20 \cdot 0.2 = \underline{4}$

$A = \underline{100}$

relative error in  $A$  is  $\frac{dA}{A} = \frac{4}{100} = 0.04$  or  $\underline{4\%}$

in  $l$  is  $\frac{dl}{l} = \frac{0.2}{10} \approx 0.02$  or  $\underline{2\%}$

Panel 5

took a quiz like Quiz 3 again,  
and also quiz #4

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Panel 6

Last time:

relative max and min

absolute max and min

Extreme Value Theorem: Every continuous function on  
a closed interval  $[a, b]$  has abs. max + min.

Fermat's Theorem: If  $f$  has rel. max or min  
at  $x=c$ , and  $f'(c)$  exists, then  $f'(c)=0$

Critical points: where  $f'=0$  or  $f'$  d.n.e.,

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Panel 7

How to find local extrema (min or max)

- 1.)  $f' = 0$  or  $f'$  d.n.e.  $\Rightarrow$  critical
- 2.) Make table using critical points
- 3.) Decide on signs of  $f'$  to conclude about  $f$ .

Ex:  $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$

1.)  $f'(x) = 12x^3 - 12x^2 - 24x = 0$   
 $= 12x(x^2 - x - 2) =$   
 $\Rightarrow = \underset{+}{12x}(x - 2)\underset{-}{(x + 1)} = 0$   
 $x = 0, x = 2, x = -1$   
 $\uparrow \quad \uparrow \quad \uparrow$   
 max    min    min

	-1	0	2
$f'$	-	+	-
$f$	$\searrow$	$\nearrow$	$\searrow$

+0.00001

Panel 8

$f(x) = x^{3/5} \cdot (4-x)$  find local extrema

$f'(x) = \frac{3}{5}x^{-2/5}(4-x) + x^{3/5}(-1) = 0$

$\frac{3}{5}x^{-2/5}(4-x) = x^{3/5} \quad | \cdot 5x^{2/5}$

$3(4-x) = 5x$

$12 - 3x = 5x$

$12 = 8x$

$\frac{12}{8} = x$  is critical.

because  $f'(0)$  d.n.e.

Also  $x=0$

	0	$3/2$
$f'$	+	+
$f$	$\nearrow$	$\searrow$

$x=0$  no extrema

$x=3/2$  is max.

Panel 9

$$\begin{aligned} \underline{f'(x)} &= \frac{3}{5} x^{-2/5} (4-x) + x^{3/5} (-1) = \\ &= \frac{3(4-x)}{5x^{2/5}} - x^{3/5} = \frac{3(4-x) - 5x}{5x^{2/5}} = \frac{12-8x}{5x^{2/5}} \end{aligned}$$

$$x = -1 : \frac{+}{-}$$

$$x^{2/5} = \sqrt[5]{x^2}$$

$$x = 1 : +$$

$$x = +10 \text{oooo} : -$$

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Panel 10

How to find absolute extrema:

Oh Handlung

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