Panel 1

Last time

Implicit diff.

! Related Rates !

Linear Approx

Differentials + Error estimates

Quiz: Derivatives < Rules / Meanings

~~today~~

Implicit diff.

1

Panel 2

Suppose the cost for producing x items is

$$C(x) = 100 - x + 2x^2$$

$C(3) = 100 - 3 + 18 = 115$

$C(2) = 100 - 2 + 8 = 106$

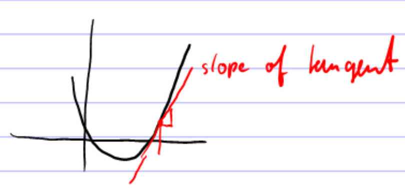
a) Fix costs: $C(0) = \$100$

b) Marginal cost: $C'(x) = \underline{-1 + 4x}$

c) If you produce 2 items, what's the additional cost to produce 1 more item, approx + exactly.

exact: $C(3) - C(2) = 9$ ← exact, but hard

$C'(2) = 7$ — quick, but approx



2

Panel 3

Quiz 3 (1)

① Calculate the derivative y' for each function (don't simplify)

a) $y = 2x\cos(x)$

b) $y = \sin\left(\sqrt{1-x}\right)$

c) $y = \dfrac{x(2x-3)}{\cos(4x)}$

3

Panel 4

Quiz 3 (2)

② Find y', assuming that $y = y(x)$, for the equation

$$x^2 \cdot \cos(y) + 2\sin(y) = xy \quad \Big| \ \tfrac{d}{dx}$$

$$\boxed{2x\cos(y) + x^2(-\sin(y)\cdot y') + 2\cos(y)\cdot y' = 1y + xy'}$$

③ A particle moves on a vertical line so that its distance function is: $d(t) = t^3 - 12t + 3$, $t \geq 0$. What is the distance when the velocity is zero?

4

Panel 5

Related Rates Guidelines

- Draw diagram including notation
- Find equation relating all variables
- Differentiate implicitly
- Substitute know quantities and solve for unknow quality.

Linear Approx Revisited:

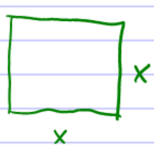
$$f'(a) = \lim_{x \to a} \frac{f(x)-f(a)}{x-a} \Rightarrow f'(a) \approx \frac{f(x)-f(a)}{x-a}$$

$$\underbrace{f'(a)}_{\text{const}}(x-a) + \underbrace{f(a)}_{\text{const}} \approx f(x) = y$$

5

Panel 6

Each side of a square is increasing at a rate of 6cm/sec. At what rate is area increasing when area is 16 cm^2.



$$A = x^2 \quad d/dt$$

$$A' = 2x x'$$

Practice:

$$A = x^2 \;\Big|\; \frac{d}{dx} \qquad\qquad A = x^2 \;\Big|\; d/dA$$

$$A' = 2x \qquad\qquad 1 = 2x \cdot x'$$

$$A' = 2x x' \qquad x = 4 \quad (A = 16)$$

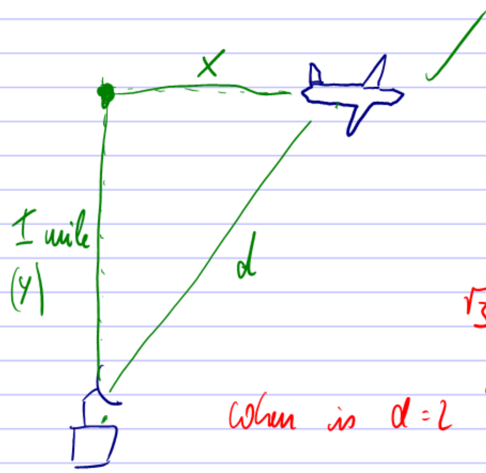
$$x' = 6$$

$$A' = 48$$

6

Panel 7

A plane flies horiz at altitude of 1 mile, speed of 500 mph. Tracked by radar station. Find how fast distance changes between plane and radar, when it is 2 miles from station.



$$x^2 + 1^2 = d^2$$

$$2xx' = 2dd'$$

$x = \sqrt{3}$, $x' = 500$, $d = 2$

$$x^2 + y^2 = d^2$$

$$2xx' + \cancel{2yy'} = 2dd'$$

$y' = 0$

when is $d = 2$? $x^2 + 1 = 2^2 = 4 \Rightarrow x = \sqrt{3}$

7

Panel 8

Linear Approx. Examples

$$f(x) \approx f'(a)(x-a) + f(a)$$

$\sin(x)$ near $a = 0$:

$\sin(x) = \cos(0)\cdot(x-0) + \sin(0)$

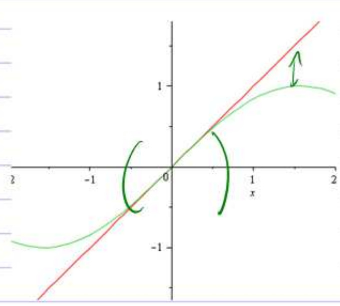
$\Rightarrow \boxed{\sin(x) = x}$ if x is small

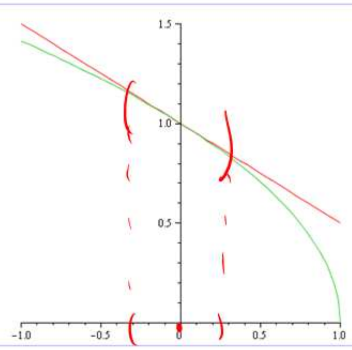
$f'(x) = \frac{1}{2}(1-x)^{-1/2}\cdot(-1)$

$\sqrt{1-x}$ near $a = 0$:

$\sqrt{1-x} \approx -\frac{1}{2}(x-0) + 1$

$\approx -\frac{1}{2}x + 1$ near 0





8

Panel 9

Last time

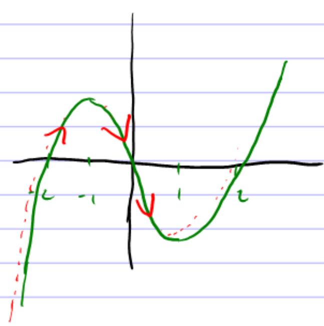
X done already

9

Panel 10

Chapter 3: Applications of Derivatives

Goal: sketch a curve quickly, including main characteristics.



increasing from x up to -1

decreasing until x = 1

increasing after x = 1

Shift happens at

x = -1 mountain top

x = 0 change steering

x = 1 valley

10

Panel 11

Terminology

A function has an absolute maximum at $x=c$ if

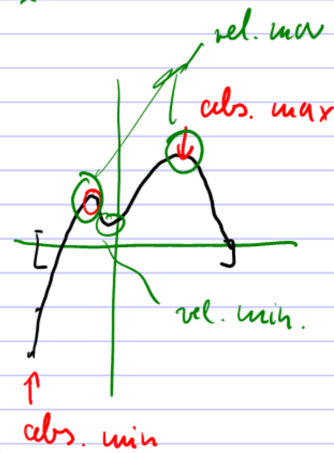
$$f(c) \geq f(x) \quad \text{for all } x$$

A function has an absolute minimum at $x=c$ if

$$f(c) \leq f(x) \quad \text{for all } x$$

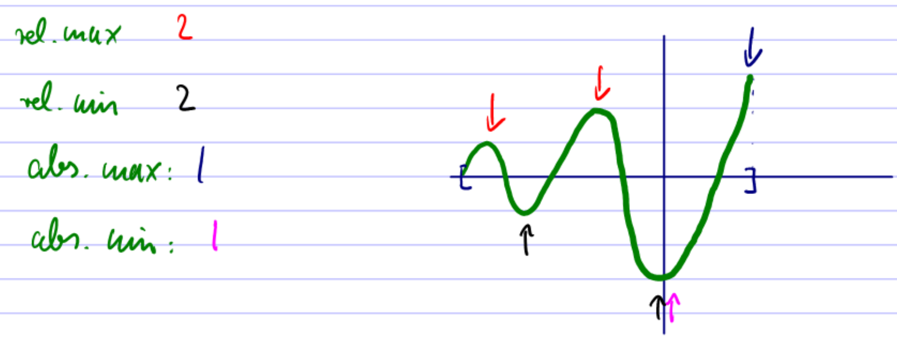
A relative max. at $x=c$ means that $f(c) \geq f(x)$ for all x near c.

Same for relative min.



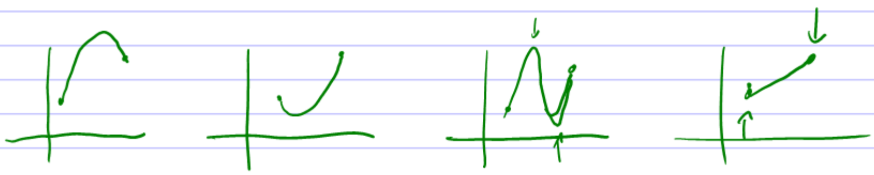
11

Panel 12



Extreme Value Theorem

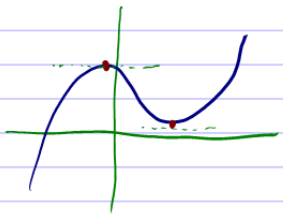
If f is continuous on closed interval $[a,b]$, then f has both abs. max and abs. min.



12

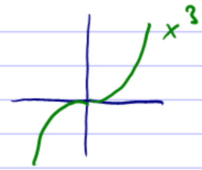
Panel 13

How to find (relative and absolute) extrema



Fermat's Theorem: If f has a rel. max or min at $x = c$, and $f'(c)$ exists, then $f'(c) = 0$

Opposite true?



$f'(0) = 0$ but no max or min!

To find rel. max/min:

- find all points where $f'(x) = 0$
- find all points where $f'(x)$ d.n.e.

$\}$ potential max/min or $\boxed{\text{critical point}}$

13

Panel 14

Example: Find all critical points for $f(x) = x^3 - 3x$

$$f'(x) = 3x^2 - 3 = 0 \Rightarrow x = +1 \text{ or } -1$$

$$3(x^2 - 1) = 0$$

$$3(x+1)(x-1) = 0$$

$\Rightarrow$ 2 critical points, and they may or may not be max/min!

14

Panel 15

Example. Find the critical points of $f(x) = x^{3/5}(4-x)$

$$f'(x) = \frac{3}{5}x^{-2/5}(4-x) - x^{3/5} = 0$$

$$\frac{3(4-x)}{5x^{2/5}} - x^{3/5} = 0$$

$$\frac{12-3x}{5x^{2/5}} = x^{3/5} \quad \Big| \cdot 5x^{2/5}$$

$$12-3x = 5x$$

$$12-8x = 0, \quad x = 12/8 = 3/2$$

critical points: $x = 3/2$, $x=0$ (f' d.n.e. at $x=0$)

15

Panel 16

How to decide if critical point is extrema

Note: If $f'(x) > 0$ then f "goes up", i.e. is increasing

If $f'(x) < 0$ then f "goes down", i.e. is decreasing

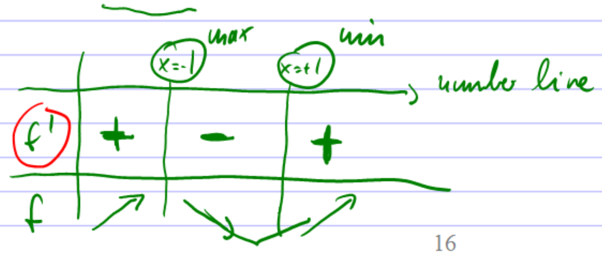
First Derivative Test:

If f changes from incr. to decreasing $\Rightarrow$ rel. max

If f changes from decr. to increasing $\Rightarrow$ rel. min.

Ex: $f(x) = x^3 - 3x \Rightarrow$ critical $x = +1, -1$,

$$f'(x) = 3x^2 - 3 = 3(x^2-1)$$



16