Panel 1

<u>Last time</u>

→ on blackboard, sorry

1

Panel 2

<u>Ex:</u> Find tangent line to $x^2 + y^2 = 2xy$ at $(2,2)$

no good.

2

Panel 3

HW . $s(t) = t^3 - 3t$

$v(t) = 3t^2 - 3$

$a(t) = 6t$

accel. when velocity is zero

$3t^2 - 3 = v(t) = 0$

$3(t^2-1) = 3(t+1)(t-1) = 0$

$\Rightarrow t = 1 \quad , \cancel{t=-1}$

$a(t=1) = 6$

3

Panel 4

48) $s(t) = 80t - 16t^2$

a) max. height?

$s'(t) = 80 - 32t = 0$ gives t of vertex

$t = 80/32 = \#$

$s(80/32) =$ max height.

Quiz Monday

4

Panel 5

$x^2 - 2xy + y^3 = c$ find dy/dx — $y = y(x)$, x variable, c constant

explicitly!

$x^2 - 2xy + y^3 = c \quad \Big| \frac{d}{dx}$

$2x - 2y - 2xy' + 3y^2 y' = 0$

$2x - 2y = 2xy' - 3y^2 y'$

$2x - 2y = y'(2x - 3y^2)$

$$\frac{2x-2y}{2x-3y^2} = y' \quad \checkmark$$

5

Panel 6

$$\frac{d}{dx}(2xy) = \frac{d}{dx}\left[(2x)\cdot(y)\right] =$$

$$= \frac{d}{dx}(2x)\cdot y + 2x \cdot \frac{d}{dx}(y) =$$

$$= 2\cdot y + 2x\, y'$$

6

Panel 7

Let x, y be functions of t such that

$$x^2 - xy + y^2 = 3 \quad \Big| \; \frac{d}{dt}$$

Find $x'(t)$, explicitly

implicit answer

$$2x \cdot x' - x'y - x \cdot y' + 2yy' = 0$$

$$2xx' - x'y = xy' - 2yy'$$

$$2xx' - x'y = y'(x - 2y)$$

$$\frac{2xx' - x'y}{x - 2y} = y'$$

7

Panel 8

Related Rates Problems

Air is pumped into a (spherical) balloon so that its volume increases at a rate of $100 cm^3/s$. How fast is the radius increasing when diameter is 50 cm?

Radius and volume are related:

$V = \frac{4}{3}\pi r^3$, where both $V = V(t)$ and $r = r(t)$

Want to know: $dr/dt = r'(t)$

$$V = \frac{4}{3}\pi r^3 \quad \Big| \; \frac{d}{dt}$$

$$\frac{dV}{dt} = \frac{4}{3}\pi 3 r^2 \cdot \frac{dr}{dt} \Rightarrow \overset{100}{\frac{dV}{dt}} = 4\pi \overset{25}{r}^2 \frac{dr}{dt}$$

want

8

Panel 9

100 25

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

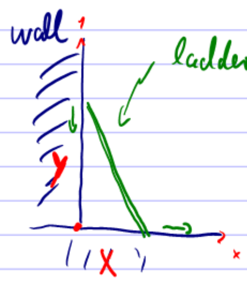
Want

$$\Rightarrow \frac{dr}{dt} = \frac{100}{4\pi\, 25^2} = ?$$

9

Panel 10

Ex: A 10 foot ladder rests against a wall. Its bottom slides away from the wall at 1 ft/sec. How fast is the top sliding down when the bottom is 6 ft from the wall?



$x^2 + y^2 = 10^2$, $\boxed{x^2 + y^2 = 100}$ $\quad d/dt$

both x, y are functions of time.

Want: dy/dt

Know: $dx/dt = 1$

$6^2 + y^2 = 100$

$y = \sqrt{64}$

$\Rightarrow 2xx' + 2yy' = 0 \quad \Rightarrow xx' + yy' = 0$

$6 + 8y' = 0 \Rightarrow y' = -\frac{6}{8}$

10

Panel 11

Related Rates Guidelines

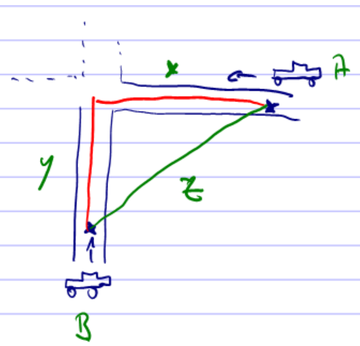
- Read the problem ✓
- Draw diagram ✓
- Introduce notation (variables) ✓
- Write down equation with variables ✓
- Implicitly differentiate
- Substitute known quantities

(Hope for the best)

11

Panel 12

Ex: Car A is traveling west at 50 mph, car B goes north at 60 mph. Both head toward intersection. At what rate are they approaching each other when A is 0.3, B is 0.4 miles from intersection.



$z^2 = x^2 + y^2 \quad \Big| \frac{d}{dt}$

$2zz' = 2xx' + 2yy' \quad \Big| \cdot \frac{1}{2}$

$zz' = xx' + yy'$

$0.25 * z' = 0.3 \cdot 50 + 0.4 \cdot 60$ ✓

$z^2 = x^2 + y^2 = 0.3^2 + 0.4^2 \Rightarrow z = 0.25$

12

Panel 13

True or False:

① If f and g are differentiable then

$$\frac{d}{dx}\left(f(x)\,g(x)\right) = f'(x)\,g'(x) \quad F$$

② If f is differentiable, then and $f(x) > 0$

$$\frac{d}{dx}\sqrt{f(x)} = \frac{f'(x)}{2\sqrt{f(x)}} \quad (T)$$

③ If f is differentiable, then

$$\frac{d}{dx} f(\sqrt{x}) = \frac{f'(\sqrt{x})}{2\sqrt{x}} \quad F$$

④ If $f'(r)$ exists, then

$$\lim_{x\to r} f(x) = f(r) \quad T$$

diff $\Rightarrow$ cont.

⑤ $\frac{d}{dx}\left[\tan^2(x)\right] = \frac{d}{dx}\left[\sec^2(x)\right]$

$2\tan\sec^2$ $\quad$ $2\sec\cdot\tan\sec$ $\quad$ True!

13

Panel 14

Other Applications of Derivatives

a) Linear approximation

Know:
$$f'(x) \approx \frac{f(x+h)-f(x)}{h} \quad (h \text{ small})$$

$$h\cdot f'(x) \approx f(x+h) - f(x)$$

$$f(x) + h\cdot f'(x) \approx f(x+h)$$

or
$$\boxed{f(x+h) \approx h\cdot f'(x) + f(x)} \quad \text{if } h \text{ small}$$

Find $\sqrt{3.98}$ consider $f(x) = \sqrt{x}$, $f'(x) = \frac{1}{2}\frac{1}{\sqrt{x}}$,

take $x = 4 \Rightarrow h = -0.02$

$$\Rightarrow \sqrt{3.98} = \sqrt{\underset{x}{4}\underset{h}{-0.02}} \approx -0.02\cdot f'(4) + f(4)$$

14

Panel 15

or $\boxed{f(x+h) \approx h \cdot f'(x) + f(x)}$ if h small

Find $\sqrt{3.98}$ consider $f(x) = \sqrt{x}$, $f'(x) = \frac{1}{2}\frac{1}{\sqrt{x}}$,

take $x = 4 \Rightarrow h = -0.02$

$$\Rightarrow \sqrt{3.98} = \sqrt{\underset{x}{4} - \underset{h}{0.02}} \approx -0.02 \cdot f'(4) + f(4)$$

$$= -0.02 \cdot 1/4 + 2 =$$

$$= -0.005 + 2 = \underline{\underline{1.995}}$$

$\sqrt{3.98} \approx 1.99499\ldots$ very good approx!

15

Panel 16

Find $\sqrt{4.05}$, approx.

$f(x) = \sqrt{x} \Rightarrow f(x+h) \approx h \cdot f'(x) + f(x)$

$x = 4$, $h = 0.05$ $\sqrt{4.05} \approx 0.05 \cdot \frac{1}{4} + 2 = \#$

Physics: $\sin(x)$ is about same as x

read in Book why that works for x small!

<u>HW</u> p. 133 - 135

16

Panel 17

5) Differentials: interprete "dx" as "small change in x" and "dy" as "change in y" and so on!

$\frac{dy}{dx}$ means deriv. of y, dy and dx differentials

Ex: Radius of sphere is 21 cm with error ± 0.05 cm (dr)

What is error in computing volume? dV

$$V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dr} = \frac{4}{3}\pi 3r^2 = 4\pi r^2$$

$$\Rightarrow dV = 4\pi r^2\, dr$$

$$\underbrace{\text{error in vol}} = \pm 4 \cdot \pi (21)^2 \cdot 0.05 = \#$$

17

Panel 18

$x^2 + 2xy - y^2 + x = 2 \qquad x=1,\ y=2$

slope of tangent

$$\frac{d}{dx}: 2x + 2xy' + 2y - 2yy' + 1 = 0$$

$$2 + 2\cdot y' + 4 - 4y' + 1 = 0$$

$$7 + -2y' = 0$$

$$7 = 2y'$$

$$7/2 = y'$$

18

Panel 19

Ex: Edge of a cube is 30cm ± 0.1cm. Find

a) max poss. error + rel. error (relative) in computing volume

b) max. poss. error + rel. error in computing surface

$V = x^3$, where x is length of one side

$\frac{dV}{dx} = 3x^2 \Rightarrow dV = 3x^2\,dx$

$\Rightarrow dV = 3 \cdot (30)^2 \cdot (\pm 0.1) = \pm 270\ cm^3$

Relative Error $\frac{df}{f}$ or in this $\frac{dV}{V} = \frac{270}{30^3} = \#$

b) left as HW

19