Panel 1

#8) $x^3 + x - 1 = 0$ has sol. in a) $[-1, 1]$ b) $[-1, 0]$

$f(-1) = -3$

~~$f(1) = +1$~~

$f(0) = -1$

?

$$f(x) = \begin{cases} \frac{x^2-4}{x-2} & \text{if } x \neq 2 \\ k & \text{if } x = 2 \end{cases}$$ cont. at $x = 2$

$\Leftrightarrow \lim_{x \to 2} f(x) = f(2) = k$

$$\lim_{x \to 2} \frac{x^2-4}{x-2} = \lim_{x \to 2} \frac{(x+2)(x-2)}{x-2} = 4 = k$$

1

Panel 2

Last time

Product Rule $\frac{d}{dx} f \cdot g = f' g + f g'$

$(f \cdot g)' = f' \cdot g + f \cdot g'$

Quotient Rule: $\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{f'g - fg'}{g^2}$

$\left(\frac{f}{g}\right)' = \frac{f' \cdot g + f \cdot g'}{g^2}$

Chain Rule:

$d/dx\, f(g(x)) = f'(g(x)) \cdot g'(x)$

$(f(g))' = f'(g(x)) \cdot g'(x)$

2

Panel 3

Derivatives of Trig Functions

(x) $\frac{d}{dx}\sin(x) = \cos(x)$ $\quad \frac{d}{dx}\cos(x) = -\sin(x)$

memory

(x) $\frac{d}{dx}\tan(x) = \frac{d}{dx}\frac{\sin(x)}{\cos(x)} = \frac{\cos^2(x)+\sin^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)} = \sec^2(x)$

(x) $\frac{d}{dx}\sec(x) = \frac{d}{dx}\frac{1}{\cos(x)} = \frac{0\cdot\cos(x)+1\cdot\sin(x)}{\cos^2(x)} = \frac{\sin}{\cos^2} = \frac{\sin}{\cos}\cdot\frac{1}{\cos}$

$= \tan(x)\sec(x)$

$\frac{d}{dx}\cot(x) = -\csc^2(x)$

$\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$

3

Panel 4

$f(x) = \tan(x^3)$

$\Rightarrow f'(x) = \sec^2(x^3)\cdot 3x^2$

$f(x) = \sec(\tan(\sin(\sqrt{x})))$

$\Rightarrow f'(x) = \sec(\tan(\sin(\sqrt{x})))\cdot\tan(\tan(\sin(\sqrt{x})))\cdot\sec^2(\sin(\sqrt{x}))\cos(\sqrt{x})\frac{1}{2}x^{-1/2}$

p 119

#11 $f(x) = \cos(a^3+x^3) \Rightarrow f'(x) = -\sin(a^3+x^3)\cdot 3x^2$

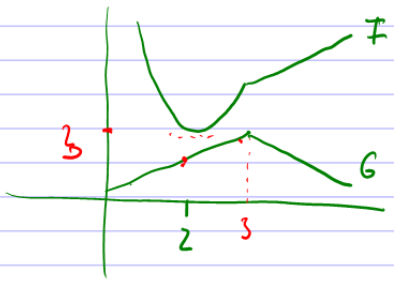
$f(a) = \cos(a^3+x^3) \Rightarrow f'(a) = -\sin(a^3+x^3)\cdot 3a^2$

(x const)

4

Panel 5

#44



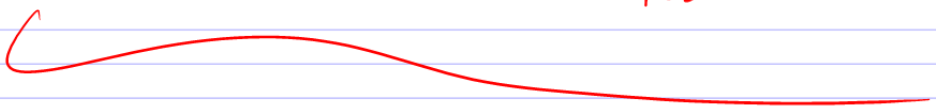
$P(x) = F(x)\,G(x)$

Sign of $P'(2)$ is:

$P'(2) = F'(2)\cdot G(2) + F(2)\,G'(2)$

$0 \cdot + \;\; + \cdot \text{pos}$

$= \text{pos}$



5

Panel 6

III

#21

$$F(x) = \frac{x - 3x\sqrt{x}}{\sqrt{x}} = \frac{x}{\sqrt{x}} - \frac{3x\sqrt{x}}{\sqrt{x}} = x^{1/2} - 3x$$

$$F'(x) = \frac{1}{2}x^{-1/2} - 3$$

$$F'(x) = \frac{\left(1 - \left(3\sqrt{x} + 3x\cdot\frac{1}{2}x^{-1/2}\right)\right)\sqrt{x} - \left(x - 3x\sqrt{x}\right)\cdot\left(\frac{1}{2}x^{-1/2}\right)}{\left(\sqrt{x}\right)^2}$$

6

Panel 7

#13 Worksheet

$f(x) = \tan(x^3)\sqrt{3-4x}$

$\Rightarrow f'(x) = (\quad A \quad)\sqrt{3-4x} + \tan(x^3)(\quad B \quad)$

$f''(x)$

$f(x) = \tan(x) \Rightarrow f'(x) = \sec^2(x)$

$f(x) = \tan(x^3) \Rightarrow f'(x) = \sec^2(x^3)\cdot 3x^2 \leftarrow A$

$f(x) = \sqrt{3-4x} = (3-4x)^{1/2} \Rightarrow f'(x) = \frac{1}{2}(3-4x)^{-1/2}\cdot(-4) \leftarrow B$

7

Panel 8

Applications of Derivative - Physics

$$\text{(avg.) speed} = \frac{\text{change in dist}}{\text{change in time}} = \frac{d(t+h)-d(t)}{h}$$

$$\text{instantaneous speed} = \lim_{h\to 0}\frac{d(t+h)-d(t)}{h} = d'(t)$$

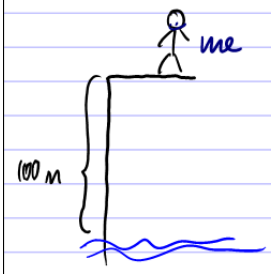
Note: The inst. velocity is derivative of distance, i.e. $v(t) = d'(t)$

Also: Derivative of velocity is acceleration.
i.e. $a(t) = v'(t) = d''(t)$

8

Panel 9

Speed and Acceleration



I drop a rock. It's distance is:

$d(t) = -4.9t^2 + 100$

How fast is rock at impact?

$\Rightarrow$ $v(t)$ when it hits the ground

$v(t) = -9.8t$

When does it hit ground?

$d(t) = 0 \iff -4.9t^2 + 100 = 0$

$t^2 = \frac{100}{4.9}$

$t = \pm\sqrt{\frac{100}{4.9}} = 4.5$

$\Rightarrow v(4.5) = -9.9 \cdot 4.5 = -44.2$ m/sec

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Panel 10

If a stone is thrown up on the moon's surface with 10 m/sec its height after t seconds is

$h(t) = 10t - 0.83t^2$

a) Find velocity after 3 sec

b) What is velocity after rising 25m?

a) $v(3) \Rightarrow v(t) = 10 - 1.66t$

$\Rightarrow v(3) = 10 - 1.66 \cdot 3 = \#$

b) $h(t) = 25 \Rightarrow 10t - 0.83t^2 = 25$

$-0.83t^2 + 10t - 25 = 0$

look up

t = quadratic formula

10

Panel 11

If $C(x)$ is a cost function, representing the cost of producing x items, then $C'(x)$ is called the marginal cost:

$C(0)$ is fixed cost, i.e. cost of producing 0 items

$C'(x)$ marginal cost, approx cost of producing one more item at level of x

Ex: $C(x) = 10000 + 5x + 0.01x^2$

Find fixed and marg. cost at production level of 500 items.

to produce 501st item costs $15 extra!

$\Rightarrow C(0) = 10000$

$C'(x) = 5 + 0.02x \Rightarrow C'(500) = 5 + 0.02 \cdot 500 = 15$

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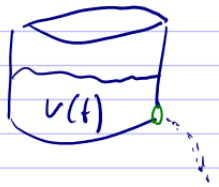
Panel 12

Torricelli's Law: If a tank holds 5000 gallons of water, which drains from the bottom of the tank in 40 minutes, then volume of water in the tank after t minutes is

$$V(t) = 5000\left(1 - \frac{t}{40}\right)^2$$



Find rate at which water drains from the tank.

$\Rightarrow$ $\frac{\text{change in volume}}{\text{change in time}} = V'(t)$

$$V'(t) = 5000 \cdot 2\left(1 - \frac{t}{40}\right)' \cdot \left(-\frac{1}{40}\right) = -\frac{5000}{40}\left(1 - \frac{t}{40}\right)$$

e.g. $V'(40) = 0$

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Panel 13

Expressions for Derivatives

$f'(x)$

$\frac{d}{dx}$

slope of tangent

(inst.) velocity

marginal cost / profit / revenue

rate of change

13

Panel 14

Implicit Differentiation

$y = \sqrt{x^3+1}$ y is an explicit function of x

$\Rightarrow y' = \frac{1}{2}(x^3+1)^{-1/2} \cdot 3x^2$

Implicitly defined function 1.) $x^2+y^2=25$

or 2) $x^3+y^3=6xy$

For 1.) Find slope of tangent at $(3,4)$

i) solve for y $\quad x^2+y^2=25$

ii) take deriv. $\quad y^2 = 25-x^2 \quad | \sqrt{\ }$

iii) subst. $\quad |y| = \pm\sqrt{25-x^2}$

$\sqrt{y^2} \quad |y|$ $\quad \Rightarrow y' = \frac{1}{2}(25-x^2)^{-1/2} \cdot (-2x)$

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Panel 15

Implicit Differentiation:

$$x^2 + y^2 = 25 \quad \Big| \frac{d}{dx}$$

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = \frac{d}{dx}(25)$$

$$2x + 2 \cdot y \cdot y' = 0$$

$$2x + 2yy' = 0 \quad \Big| \cdot \frac{1}{2}$$

$$x = -yy'$$

$$-\frac{x}{y} = y'$$

at $(3,4)$ $y' = -\frac{3}{4}$

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Panel 16

Implicitly differentiate

$$x^3 + y^3 = 6xy$$

where y is a function of x, i.e. $y = y(x)$

$$x^3 + y^3 = 6xy \quad \Big| \cdot \frac{d}{dx}$$

$$3x^2 + 3y^2 \cdot y' = 6y + 6x\,y'$$

$$3x^2 - 6y = 6xy' - 3y^2 y'$$

$$3x^2 - 6y = (6x - 3y^2)y'$$

$$\frac{3x^2 - 6y}{6x - 3y^2} = y'$$

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Panel 17

Let $\sin(x+y) = y^2\cos(x)$ where $y = y(x)$.

Find y'.

$$\sin(x+y) = y^2\cos(x) \quad \Big| \cdot \frac{d}{dx}$$

$$\cos(x+y)\cdot(1+y') = 2y\cdot y'\cdot\cos(x) + y^2(-\sin(x))$$

Next, solve for y':

$y' =$ HW

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Panel 18

Find y'' if $x^4 + y^4 = 16$

$$x^4 + y^4 = 16 \quad \Big| \frac{d}{dx}$$

$$4x^3 + 4y^3\cdot y' = 0$$

$$y' = -\frac{x^3}{y^3}$$

$$y'' = -\frac{3x^2y^3 - x^3\cdot 3y^2y'}{y^6} =$$

$$= -\frac{3x^2y^3 - x^3\cdot 3y^2\cdot\left(-\frac{x^3}{y^3}\right)}{y^6} = \text{simplify}$$

18

Panel 19

Consider $xy = 1$.

a) Find y' if $y = y(x)$

$$xy = 1 \quad \Big| \frac{d}{dx}$$

$$1 \cdot y + x y' = 0 \Rightarrow y + x y' = 0$$

b) Find x' if $x = x(y)$

$$x \cdot y = 1 \quad \Big| \frac{d}{dy}$$

$$x' \cdot y + x \cdot 1 = 0 \Rightarrow x' y + x = 0$$

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Panel 20

c) $xy = 1$ Find x' if $x = x(t)$, $y = y(t)$

$$xy = 1 \quad \Big| \frac{d}{dt}$$

$$x' y + x \cdot y' = 0$$

More on HW.

posted soon!

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