Panel 1

Last time:

Intermediate Value Theorem

Limits involving infinity
- vertical asymptote
- horizontal asymptote

Limits at infinity of rational functions

1

Panel 2

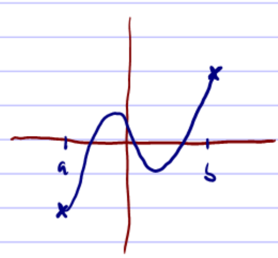
Most common IVT application

Suppose f is continuous on (a,b) and

$f(a) < \underset{m}{0}$ and $f(b) > \underset{M}{0}$ (or vice versa)

Then $\boxed{f(c) = 0}$ for at least on c in (a,b)

$f(c)$ between m and M



Ex: $\cos(x) = x$ on $(\overset{b}{0},1)$

IVT: Let $f(x) = \cos(x) - x$

Then $f(0) = \cos(0) - 0 = 1 > 0$

$f(1) = \cos(1) - 1 < 0$

also f is continuous $\Rightarrow$ by IVT there is c with $f(c) = 0$

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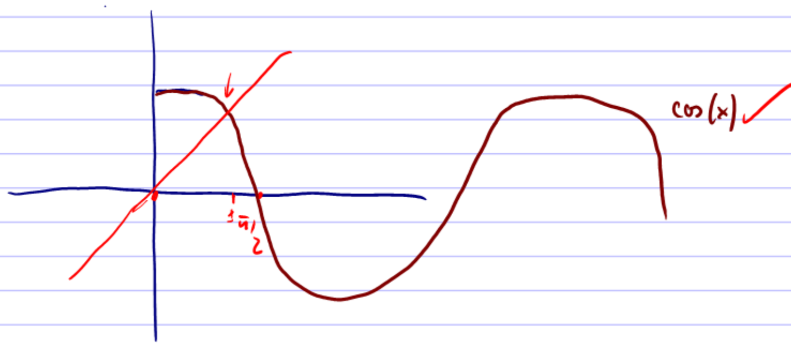
Panel 3

$\cos(x) = \boxed{x}$



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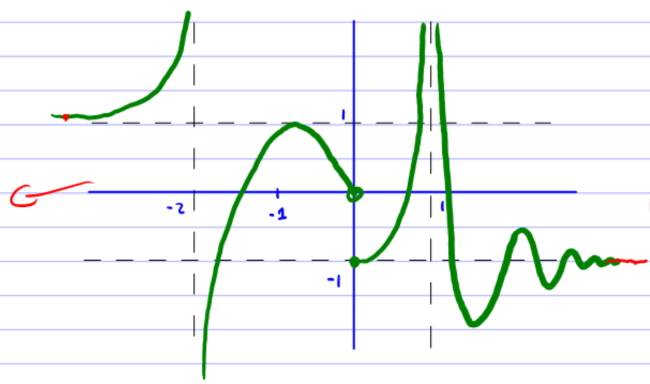
Panel 4

Limits Graphically



10) $\lim_{x \to \infty} f(x) = -1$

11) $\lim_{x \to -\infty} f(x) = 1$

12) $\lim_{x \to -1} f(x) = 1$

13) $\lim_{x \to -1^-} f(x) = 1$

1) $\lim_{x \to -2^-} f(x)$ $+\infty$

2) $\lim_{x \to -2^+} f(x)$ $-\infty$

3) $\lim_{x \to -2} f(x)$ d.n.e.

4) $\lim_{x \to 1^-} f(x)$ $+\infty$

5) $\lim_{x \to 1^+} f(x)$ $+\infty$

6) $\lim_{x \to 1} f(x)$ $+\infty$

7) $\lim_{x \to 0^-} f(x) = 0$

8) $\lim_{x \to 0^+} f(x) = -1$

9) $\lim_{x \to 0} f(x)$ d.n.e

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Panel 5

Horizontal Asymptotes of Rational Functions

$\lim_{x \to \infty} \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials

When in doubt, factor out (highest power)

$$\lim_{x \to \pm\infty} \frac{p(x)}{q(x)} = \begin{cases} \# & \text{if } \operatorname{degree}(p) = \operatorname{degree}(q) \\ 0 & \text{if } \operatorname{degree}(p) < \operatorname{degree}(q) \\ \pm\infty & \text{if } \operatorname{degree}(p) > \operatorname{degree}(q) \end{cases}$$

$\lim_{x \to \infty} \frac{x^4 + x^4}{7x^5 + x}$

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Panel 6

Limits involving infinity are based on:

$\lim_{x \to \infty} \frac{1}{x} = \frac{1}{\infty} = 0$

$\lim_{x \to 0^+} \frac{1}{x} = \frac{1}{+0} = +\infty$

$\lim_{x \to -\infty} \frac{1}{x} = \frac{1}{-\infty} = 0$

$\lim_{x \to 0^-} \frac{1}{x} = \frac{1}{-0} = -\infty$

$\lim_{x \to 0} \frac{1}{x} = $ d.n.e.

$\frac{\infty}{\infty}$ or $\frac{0}{0}$ require more work!

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Panel 7

Horizontal Asymptotes of Rational Functions

$\lim\limits_{x \to \infty} \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials

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Panel 8

Gravitational Force

$$F(r) = \begin{cases} \frac{GMr}{R^3}, & r < R \\ \frac{GM}{r^2}, & r \geq R \end{cases}$$

Question: is F continuous?

$$(=) \quad \lim_{r \to R} F(r) : \begin{cases} \lim\limits_{r \to R^+} F(r) = \lim\limits_{r \to R^+} \frac{GM}{r^2} = \frac{G \cdot M}{R^2} \\ \lim\limits_{r \to R^-} F(r) = \lim\limits_{r \to R^-} \frac{GMr}{R^3} = \frac{GM\cancel{R}}{R^{\cancel{3}\,2}} = \frac{GM}{R^2} \end{cases}$$

$$= \frac{GM}{R^2} = F(R)$$

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Panel 9

37) $x^4+x-3=0$ has a solution on $(1,2)$

1.) x^4+x-3 is cont. on $(1,2)$
2.) $f(1) = -1 < 0$
$f(2) = 15 > 0$

Thus, there is c s.t. $f(c)=0$ ✓

36) Prove that there is a number such that $c^2=2$

$f(x) = x^2 - 2 = 0$?

$f(1) < 0$
$f(2) > 0$
$\Rightarrow$ there is a number between 1 and 2 with $x^2-2=0$

in fact, $x = \sqrt{2} = 1.414\ldots$

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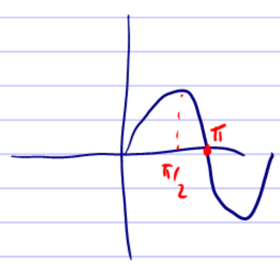
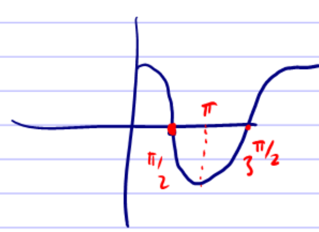
Panel 10

16) $$\lim_{x\to\pi^-} \tan(x) = \lim_{x\to\pi^-} \frac{\sin(x)}{\cos(x)} = \frac{0}{-1} = 0$$

$$\lim_{x\to\pi^-} \cot(x) = \lim_{x\to\pi^-} \frac{\cos(x)}{\sin(x)} = \frac{-1}{+0} = -\infty$$





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Panel 11

27) $\lim_{x\to\infty}\left(x-\sqrt{x}\right) = \infty - \infty = \lim_{x\to\infty}\left(x-\sqrt{x}\right)\frac{x+\sqrt{x}}{x+\sqrt{x}} =$

$= \lim_{x\to\infty}\frac{x^2-x}{x+\sqrt{x}} = \lim_{x\to\infty}\frac{x^2\left(1-\overset{\to 0}{\frac{1}{x}}\right)}{x\left(1+\sqrt{\underset{\to 0}{\frac{1}{x}}}\right)} = \lim_{x\to\infty} x\cdot 1 = \infty$

28) $\lim_{x\to\infty}\frac{x^3-2x+3}{5-2x^3} = \frac{\infty}{\infty} = -\frac{1}{2}$ ✓

$\infty - \infty\ ?$ $\qquad \infty + \infty = \infty$

$+\infty + \infty\ ?$ $\qquad -\infty - \infty = -\infty$

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Panel 12

a) $\lim_{x\to\infty}\frac{x^3-5x}{2-x^4} = 0$

b) $\lim_{x\to-\infty}\frac{x^3-5x}{2-x^2} = \lim_{x\to-\infty}\frac{x^3\left(1-5/x^2\right)}{x^2\left(2/x^2-1\right)} = \lim_{x\to-\infty} x\,(-1) = +\infty$

$\lim_{x\to-\infty}\frac{x^3-5x}{2-x} = \lim_{x\to-\infty}\frac{x^3\left(1-5/x^2\right)}{x\left(2/x-1\right)} = \lim_{x\to-\infty} x^2(-1) = -\infty$

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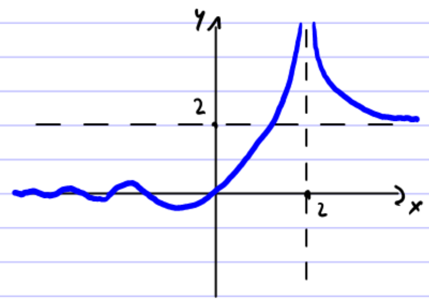
Panel 13

Quiz #2 Name: __________

① Show that $x^4 - x^2 - 1 = 0$ has at least one solution in the interval $(0, 2)$

② Consider the function shown and find the given limits



a) $\lim_{x \to 2^+} f(x)$

b) $\lim_{x \to \infty} f(x)$

c) $\lim_{x \to -\infty} f(x)$

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Panel 14

③ Find the following limits:

a) $\lim_{x \to 2^-} \frac{3x+1}{x-2}$

b) $\lim_{x \to \infty} \frac{3 - x^2}{1 + x + 2x^2}$

c) $\lim_{x \to -\infty} \frac{5x^3 + 7x - 9}{6x^5 + 3x^3 - 7x}$

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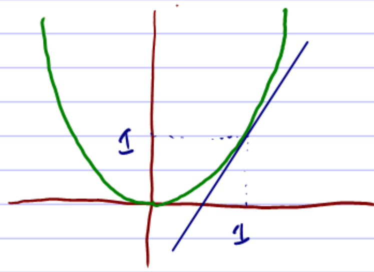
Panel 15

Done with limits. Next we'll do more limits!

Tangent problem: Find tangent line to $y = x^2$ at $x = 1$



tangent line is a line that just "touches" the graph at that point

Ex: tangent line at $x=0$ has equation $y=0$

tangent line at $x=1$ has positive slope

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Panel 16

~~Chapter 2: Differentiation~~

~~Let~~ $\cancel{f(x) = 2x^2+1}$, ~~find~~ $\cancel{\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$

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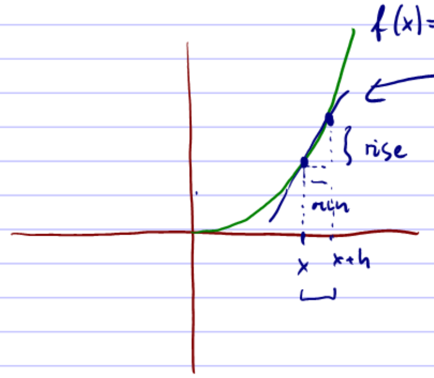
Panel 17



slope of this line is

$$\lim_{h \to 0} \frac{f(x+h)-f(x)}{(x+h)-x}$$

Def: Slope of tangent line to a function f is:

$$\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

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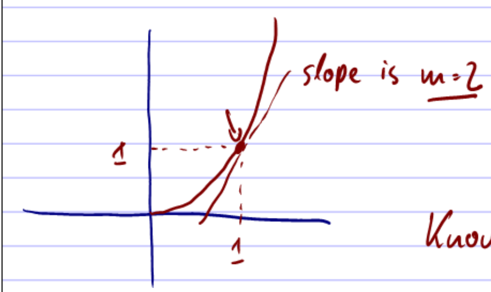
Panel 18

In our case: $f(x) = x^2$, at $x=1$

slope is $\lim_{h \to 0} \frac{f(1+h)-f(1)}{h} = \lim_{h \to 0} \frac{(1+h)^2-1}{h} =$

$$= \lim_{h \to 0} \frac{1+2h+h^2-1}{h} = \lim_{h \to 0} \frac{\cancel{h}(2+h)}{\cancel{h}} = 2$$



Know: $f(x) = mx+b =$
$= 2x+b$

Know line goes through $x=1, y=1$

$\Rightarrow f(1) = 1 = 2 \cdot 1 + b, \Rightarrow b = -1$

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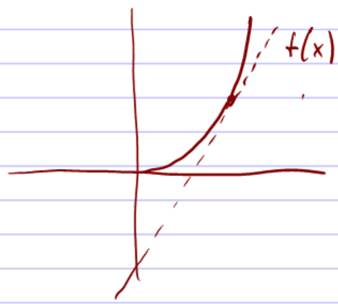
Panel 19



The limit $\lim\limits_{h\to 0} \frac{f(x+h)-f(x)}{h}$ has a special

name: slope of tangent

derivative of a function $f(x)$

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Panel 20

Find the derivative of $f(x) = x^2 - 8x + 9$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\left((x+h)^2 - 8(x+h) + 9\right) - \left(x^2 - 8x + 9\right)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{8x} - 8h + \cancel{9} \cancel{-x^2} + \cancel{8x} - \cancel{9}}{h}$$

$$= \lim_{h\to 0} \frac{2xh + h^2 - 8h}{h} = \lim_{h\to 0} \frac{\cancel{h}(2x+h-8)}{\cancel{h}} = \underline{\underline{2x-8}}$$

Goal: Find quick ways to compute the Derivative.

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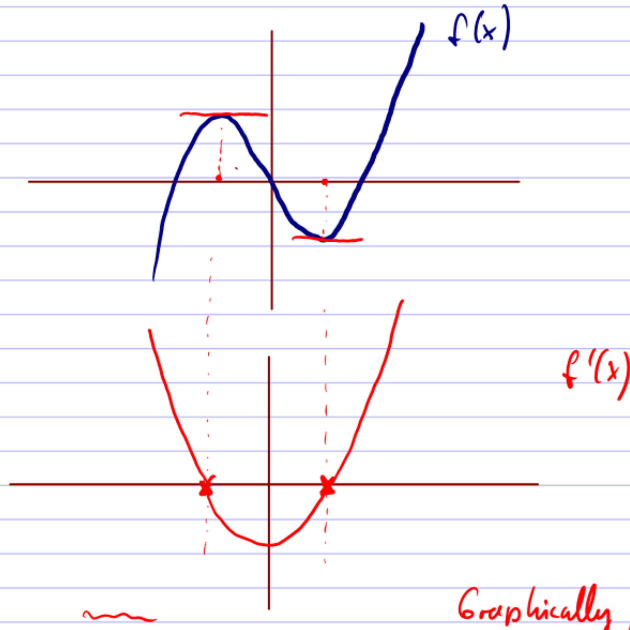
Panel 21



Graphically, f' is "easy" to see!

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Panel 22

How to compute derivative quickly algebraically:

Next time!

will post HW

Ex I on Monday

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