Panel 1

Last time:

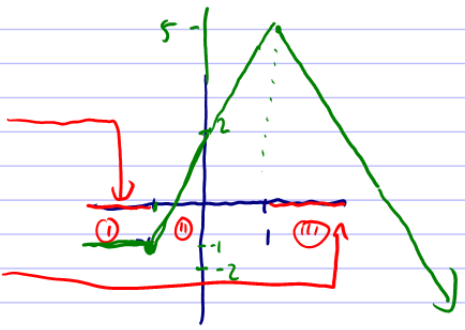
Function, domain, range

Piecewise defined function

Linear, power, exp, log, poly, trig, rational.

page 9 #40:

$$f(x) = \begin{cases} -1 & x \le -1 \\ 3x+2 & |x| < 1 \\ 7-2x & x \ge 1 \end{cases}$$



Domain: $\mathbb{R}$

Range: $(-\infty, 5]$

1

Panel 2

What about $|x|$?

$$|x| = \begin{cases} x & \text{if } x \ge 0 \\ -x & \text{if } x < 0 \end{cases}$$

$|x| < 1 \iff -1 < x < 1$

2

Panel 3

#12) $2200 for 100 chairs and
$4800 for 300 chairs
Express cost as function of chairs, using linear model

$f(x) = mx + b$, $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4800 - 2200}{300 - 100}$

x = # of chairs

blah ....

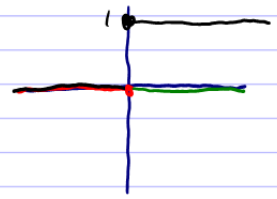
Quiz on Wed!

3

Panel 4

Heavy side function:

$$H(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$$

a) 

b) $V(t)$ voltage s.t. when switched on at $t = 0$ c) 120 Volts

$V(t) = 120\,H(t)$

c) 240 V at $t = 5$

$V(t) = 240 \cdot H(t-5)$

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Panel 5

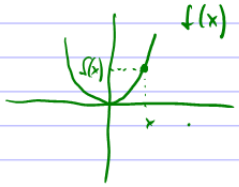
Shifting: Suppose we know graph of $f(x)$.

a) $f(x)+c$ - shift up by c

$f(x)-c$: - shift down by c

$f(x+c)$: - shift to left !

$f(x-c)$: - shift to right !



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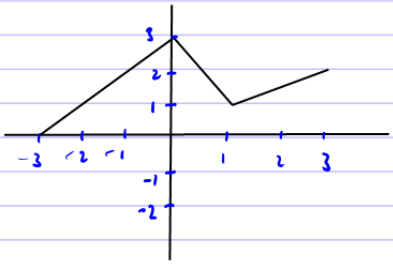
Panel 6

Graph of $f(x)$



Domain:

Range:

Let $g(x) = f(x) - 1$, $h(x) = f(x-1)$

What is: $g(0) = 2$ $h(0) = f(-1) = 2$ ✓

$g(1) = 0$ $h(2) = f(1) = 1$

g is f shifted: down

h is f shifted: right

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Panel 7

Shifting: Suppose we know graph of $f(x)$.

$\Rightarrow c\,f(x)$: stretch vertically ↕

$\frac{1}{c}\,f(x)$: compress vertically ↕

too Hard

$f(cx)$: compress horiz.

$f(\frac{1}{c}x)$: stretches horiz.

where $c > 1$

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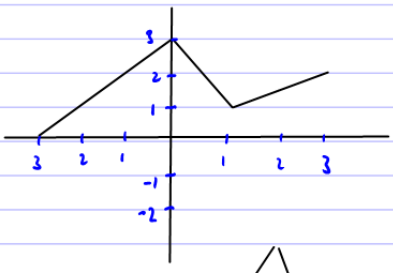
Panel 8

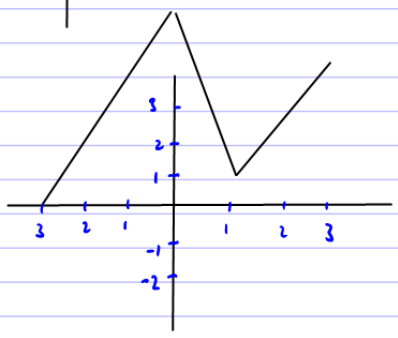
Graph of $f(x)$



Domain:

Range:

$2f(x)$:



$-f(x)$ – flip around x-axis

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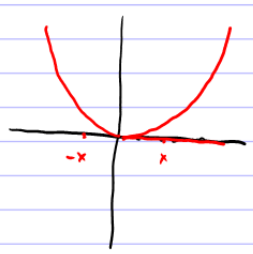
Panel 9

Shifting, Stretching, Symmetry

Symmetry:

If $f(x) = f(-x)$, f is even



If $f(x) = -f(-x)$, f is odd

Is $f(x) = x^4 + 2x^2 - 9$ even/odd?

Check $f(-x) = (-x)^4 + 2(-x)^2 - 9 =$

$= x^4 + 2x^2 - 9 = f(x)$ even

How about: $h(x) = -3x^5 + 2x^3 + 1$

how odd

$h(-x) = -3(-x)^5 + 2(-x)^3 + 1 =$

$= 3x^5 - 2x^3 + 1 = -(-3x^5 + 2x^3 - 1) = -h(x)$

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Panel 10

Limits

Say $f(x)$ is undefined for some value of x, say at $x = 3$. What if x is close to 3

$$f(x) = \frac{x^2 - 9}{x - 3}$$

x	f(x)
2.9	5.9
2.99	5.99
2.999	5.999
⋮	

What if x is close to 3?

$f(x)$ gets close to 6!

What is $f(3)$ undefined

$$\lim_{x \to 3} f(x) = 6$$

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Panel 11

Limit Notation

If $f(x)$, as x approaches a, gets closer and closer to L we write

$$\lim_{x \to a} f(x) = L$$

↑ limit as x approaches a

Ex: $\lim_{x \to -2} x^2 = 4$

$\lim_{x \to 2} (x^2 - x + 2) = 4$

$\lim_{x \to 0} \frac{x-1}{x^2-1} = 1$, $\lim_{x \to 1} \frac{x-1}{x^2-1} = \lim_{x \to 1} \frac{x-1}{(x+1)(x-1)} = \frac{1}{2}$

or make table

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Panel 12

Complicated Limits

$\lim_{x \to 1} \frac{\sin(x)}{x} = \frac{\sin(1)}{1} = \sin(1)$

$\lim_{x \to 0} \frac{\sin(x)}{x} = 1$

x	$\sin(x)$
0.001	0.999...

$\lim_{x \to 0} \sin\left(\frac{\pi}{x}\right) = \not\exists$ not exist

x	$f(x)$
0.001	0

x
0.00781	4

if $x = \frac{1}{n}$ then $\sin\left(\frac{\pi}{x}\right) = \sin\left(\frac{\pi}{1/n}\right) = \sin(n\pi) = 0$

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Panel 13

Calculating Limits

Theorem: (Plug + Chug) The limit of a $\begin{matrix}\text{diff}\\ \text{sum}\\ \text{prod}\end{matrix}\Big/\begin{matrix}\text{quot.}\\ \text{sqrt}\end{matrix}$ is the $\begin{matrix}\text{diff}\\ \text{sum}\\ \text{prod}\end{matrix}\Big/$ of the limits

quot. if denom. not equal to 0

sqrt. if non-negative

Plug + chug !!!

Find the following limits:

$$\lim_{x\to 5}(2x^2-3x+4) = 50 - 15 + 4$$

$$\lim_{x\to -2}\frac{x^3+2x^2-1}{5-3x} = \frac{-8+8-1}{5+6} = -\frac{1}{11}$$

$$\lim_{t\to 0}\sqrt{t^2+9} = \sqrt{9} = 3$$

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Panel 14

Tricky vs. Simple Limits

$$\lim_{x\to 0}\frac{x^2-1}{x-1} = \text{easy} = 2$$

$$\lim_{x\to 1}\frac{x^2-1}{x-1} = \frac{0}{0} = \lim_{x\to 0}\frac{(x+1)(x-1)}{x-1}$$

$$\lim_{h\to 0}\frac{(3+h)^2-9}{h} = \frac{0}{0} = \lim_{h\to 0}\frac{9+6h+h^2-9}{h} = \lim_{h\to 0}\frac{6h+h^2}{h} =$$

$$= \lim_{h\to 0}\frac{h(6+h)}{h} = 6$$

$$\lim_{t\to 0}\frac{(\sqrt{t^2+9}-3)}{t^2}\cdot\frac{(\sqrt{t^2+9}+3)}{(\sqrt{t^2+9}+3)} =$$

$$= \lim_{t\to 0}\frac{t^2+9-9}{t^2(\sqrt{t^2+9}+3)} =$$

$$\lim_{x\to 0}\frac{\sin(x)}{x}$$

$(a-b)(a+b) = a^2-b^2$

$$= \lim_{t\to 0}\frac{t^2}{t^2(\sqrt{t^2+9}+3)} = \frac{1}{6}$$

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Panel 15

Limits of Piecewise defined Functions:

$$H(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$$

$\lim_{x \to 2} H(x) = 1$

$\lim_{x \to -2} H(x) = 0$ $\quad x = 0.0001 \Rightarrow H(x) = 1$

$\lim_{x \to 0} H(x) =$ ☹ undefined d.n.e. $\quad x = -0.0001 \Rightarrow H(x) = 0$

∅

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Panel 16

One-Sided Limits

$\lim_{x \to a^+} f(x)$ is limit as $x \to a$ but $x > a$

$\lim_{x \to a^-} f(x)$ is limit as $x \to a$ but $x < a$

Ex: $f(x) = \begin{cases} x^2 - 1 & x < 0 \\ 2x + 1 & x \geq 0 \end{cases}$

$\lim_{x \to 0} f(x)$ (if $x > 0$) $= 1 = \lim_{x \to 0^+} f(x) = 1$

$\lim_{x \to 0} f(x)$ (if $x < 0$) $= -1 = \lim_{x \to 0^-} f(x) = -1$

$\Rightarrow \lim_{x \to 0} f(x) =$ DNE

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