A system of linear equations can be solved with the method of elimination by addition or elimination by substitution. A solution may involve one or more parameters. Substitution is also useful in solving nonlinear systems.

Solving a system formed by the supply and demand equations for a product gives the equilibrium point, which indicates the price at which consumers will purchase the same quantity of a product that producers wish to sell at that price.

Profit is total revenue minus total cost, where total cost is the sum of fixed costs and variable costs. The break-even points are the points where total revenue equals total cost.

Review Problems

Problem numbers shown in color indicate problems suggested for use as a practice chapter test.

1. The slope of the line through (2, 5) and (3, k) is 4. Find k.

2. The slope of the line through (5, 4) and (k, 4) is 0. Find k.

In Problems 3–9, determine the slope-intercept form and a general linear form of an equation of the straight line that has the indicated properties.

3. Passes through $(-2, 3)$ and has y-intercept -1

4. Passes through $(-1, -1)$ and is parallel to the line $y = 3x - 4$

5. Passes through (8, 3) and has slope 3

6. Passes through (3, 5) and is vertical

7. Passes through $(-2, 4)$ and is horizontal

8. Passes through (1, 2) and is perpendicular to the line $-3y + 5x = 7$

9. Has y-intercept -3 and is perpendicular to $2y + 5x = 2$

10. Determine whether the point (3, 11) lies on the line through (2, 7) and (4, 13).

In Problems 11–16, determine whether the lines are parallel, perpendicular, or neither.

11. $x + 4y + 2 = 0, \quad 8x - 2y - 2 = 0$

12. $y - 2 = 2(x - 1), \quad 2x + 4y - 3 = 0$

13. $x - 3 = 2(y + 4), \quad y = 4x + 2$

14. $2x + 7y - 4 = 0, \quad 6x + 21y = 90$

15. $y = 5x + 2, \quad 10x - 2y = 3$

16. $y = 7x, \quad y = 7$

In Problems 17–20, write each line in slope-intercept form, and sketch. What is the slope of the line?

17. $3x - 2y = 4$

18. $x = -3y + 4$

19. $4 - 3y = 0$

20. $3x - 5y = 0$

In Problems 21–30, graph each function. For those that are linear, give the slope and the vertical-axis intercept. For those that are quadratic, give all intercepts and the vertex.

21. $y = f(x) = 17 - 5x$

22. $s = g(t) = 5 - 3t + t^2$

23. $y = f(x) = 9 - x^2$

24. $y = f(x) = 3x - 7$

25. $y = h(t) = 3 + 2t + t^2$

26. $y = k(t) = -3 - 3t$

27. $p = g(t) = -7t$

28. $y = F(x) = (2x - 1)^2$

29. $y = F(x) = -(x^2 + 2x + 3)$

30. $y = f(x) = 5x + 2$

In Problems 31–44, solve the given system.

31. $\begin{cases} 2x - y = 6 \\ 3x + 2y = 5 \end{cases}$

32. $\begin{cases} 8x - 4y = 7 \\ y = 2x - 4 \end{cases}$

33. $\begin{cases} 7x + 5y = 5 \\ 6x + 5y = 3 \end{cases}$

34. $\begin{cases} 2x + 4y = 8 \\ 3x + 6y = 12 \end{cases}$

35. $\begin{cases} \frac{1}{2}x - \frac{1}{3}y = 2 \\ \frac{3}{4}x + \frac{1}{2}y = 3 \end{cases}$

36. $\begin{cases} \frac{1}{3}x - \frac{1}{4}y = \frac{1}{12} \\ \frac{4}{3}x + 3y = \frac{5}{3} \end{cases}$

37. $\begin{cases} 3x - 2y + z = -2 \\ 2x + y + z = 1 \\ x + 3y - z = 3 \end{cases}$

38. $\begin{cases} 2x + \dfrac{3y + x}{3} = 9 \\ y + \dfrac{5x + 2y}{4} = 7 \end{cases}$

39. $\begin{cases} x^2 - y + 5x = 2 \\ x^2 + y = 3 \end{cases}$

40. $\begin{cases} y = \dfrac{3}{x + 2} \\ x + y - 2 = 0 \end{cases}$

41. $\begin{cases} x + 2z = -2 \\ x + y + z = 5 \end{cases}$

42. $\begin{cases} x + y + z = 0 \\ x - y + z = 0 \\ x + z = 0 \end{cases}$

43. $\begin{cases} x - y - z = 0 \\ 2x - 2y + 3z = 0 \end{cases}$

44. $\begin{cases} 2x - 5y + 6z = 1 \\ 4x - 10y + 12z = 2 \end{cases}$

45. Suppose a and b are linearly related so that $a = 0$ when $b = -3$ and $a = 3$ when $b = -5$. Find a general linear form of an equation that relates a and b. Also, find a when $b = 3$.

46. Temperature and Heart Rate When the temperature, T (in degrees Celsius), of a cat is reduced, the cat's heart rate, r (in beats per minute), decreases. Under laboratory conditions, a cat at a temperature of 36°C had a heart rate of 206, and at a temperature of 30°C its heart rate was 122.
If r is linearly related to T, where T is between 26 and 38, (a) determine an equation for r in terms of T, and (b) determine the cat's heart rate at a temperature of 27°C.

47. Suppose f is a linear function such that $f(1) = 5$ and $f(x)$ decreases by four units for every three-unit increase in x. Find $f(x)$.

48. If f is a linear function such that $f(-1) = 8$ and $f(2) = 5$, find $f(x)$.

49. Maximum Revenue The demand function for a manufacturer's product is $p = f(q) = 200 - 2q$, where p is the price (in dollars) per unit when q units are demanded. Find the level of production that maximizes the manufacturer's total revenue, and determine this revenue.

50. Sales Tax The difference in price of two items before a 7% sales tax is imposed is \$2.00. The difference in price after the sales tax is imposed is allegedly \$3.10. Show that this scenario is not possible.

51. Equilibrium Price If the supply and demand equations of a certain product are $120p - q - 240 = 0$ and $100p + q - 1200 = 0$, respectively, find the equilibrium price.

52. Psychology In psychology, the term *semantic memory* refers to our knowledge of the meaning and relationships of words, as well as the means by which we store and retrieve such information.[6] In a network model of semantic memory, there is a hierarchy of levels at which information is stored. In an experiment by Collins and Quillian based on a network model, data were obtained on the reaction time to respond to simple questions about nouns. The graph of the results shows that, on the average, the reaction time, R (in milliseconds), is a linear function of the level, L, at which a characterizing property of the noun is stored. At level 0, the reaction time is 1310; at level 2, the reaction time is 1460. (a) Find the linear function. (b) Find the reaction time at level 1. (c) Find the slope and determine its significance.

53. Break-Even Point A manufacturer of a certain product sells all that is produced. Determine the break-even point if the product is sold at \$16 per unit, fixed cost is \$10,000, and variable cost is given by $y_{VC} = 8q$, where q is the number of units produced (y_{VC} expressed in dollars).

54. Temperature Conversion Celsius temperature, C, is a linear function of Fahrenheit temperature, F. Use the facts that 32°F is the same as 0°C and 212°F is the same as 100°C to find this function. Also, find C when $F = 50$.

55. Pollution In one province of a developing nation, water pollution is analyzed using a supply-and-demand model. The *environmental supply equation* $L = 0.0183 - \dfrac{0.0042}{p}$ describes the levy per ton, L (in dollars), as a function of total pollution, p (in tons per square kilometer), for $p \geq 0.2295$. The *environmental demand equation,* $L = 0.0005 + \dfrac{0.0378}{p}$, describes the per-ton abatement cost as a function of total pollution for $p > 0$. Find the expected equilibrium level of total pollution to two decimal places.[7]

56. Graphically solve the linear system

$$\begin{cases} 3x + 4y = 20 \\ 7x + 5y = 64 \end{cases}$$

57. Graphically solve the linear system

$$\begin{cases} 0.3x - 0.4y = 2.5 \\ 0.5x + 0.7y = 3.1 \end{cases}$$

Round x and y to two decimal places.

58. Graphically solve the nonlinear system

$$\begin{cases} y = \dfrac{3}{7x} & \text{where } x > 0 \\ y = x^2 - 9 \end{cases}$$

Round x and y to two decimal places.

59. Graphically solve the nonlinear system

$$\begin{cases} y = x^3 + 1 \\ y = 2 - x^2 \end{cases}$$

Round x and y to two decimal places.

60. Graphically solve the equation

$$x^2 + 4 = x^3 - 3x$$

by treating it as a system. Round x to two decimal places.

EXPLORE & EXTEND Mobile Phone Billing Plans

Selecting a mobile phone plan can be quite difficult. In most urban areas there are many service providers each offering a number of plans. The plans can include monthly access fees, free minutes, charges for additional airtime, regional roaming charges, national roaming charges, peak and off-peak rates, and long-distance charges (not to mention activation fees, cancellation fees, and the like). Even if a consumer has a fairly good knowledge of her typical mobile phone usage, she may have to do dozens of calculations to be absolutely sure of getting the best deal in town.

Mathematical modeling often involves making informed decisions about which factors in a problem are less important. These are then ignored to get a reasonably good approximate solution—in a reasonable amount of time. You may have heard the expression "simplifying assumptions." There are a lot of old jokes about this process. For example, a mathematically minded bookie who is trying to calculate the attributes of the horses in a given race should probably not assume that all the horses are perfectly spherical. We will simplify our comparison of mobile phone plans by considering just the number of "monthly home airtime minutes" available for the "monthly access fee" and the price per minute of "additional minutes." Many providers offer plans in terms of these basic parameters.

Examining Verizon's offerings for the Saddle River, New Jersey, area, in the spring of 2006, we found these America's Choice monthly plans:

P_1: 450 minutes for \$39.99 plus \$0.45 per additional minute

P_2: 900 minutes for \$59.99 plus \$0.40 per additional minute

P_3: 1350 minutes for \$79.99 plus \$0.35 per additional minute

[6] G. R. Loftus and E. F. Loftus, *Human Memory: The Processing of Information* (New York: Lawrence Erlbaum Associates, Inc., distributed by the Halsted Press, Division of John Wiley & Sons, Inc., 1976).

[7] See Hua Wang and David Wheeler, "Pricing Industrial Pollution in China: An Economic Analysis of the Levy System," World Bank Policy Research Working Paper #1644, September 1996.