d. Find the output required to obtain a profit of \$10,000.

Solution: In order to obtain a profit of \$10,000, we have

$$\begin{aligned} \text{profit} &= \text{total revenue} - \text{total cost} \\ 10{,}000 &= 8q - \left(\frac{22}{9}q + 5000\right) \\ 15{,}000 &= \frac{50}{9}q \\ q &= 2700 \end{aligned}$$

Thus, 2700 units must be produced.

Now Work Problem 9 ◁

EXAMPLE 4 Break-Even Quantity

Determine the break-even quantity of XYZ Manufacturing Co., given the following data: total fixed cost, \$1200; variable cost per unit, \$2; total revenue for selling q units, $y_{\text{TR}} = 100\sqrt{q}$.

Solution: For q units of output,

$$\begin{aligned} y_{\text{TR}} &= 100\sqrt{q} \\ y_{\text{TC}} &= 2q + 1200 \end{aligned}$$

Equating total revenue to total cost gives

$$\begin{aligned} 100\sqrt{q} &= 2q + 1200 \\ 50\sqrt{q} &= q + 600 \qquad \text{dividing both sides by 2} \end{aligned}$$

Squaring both sides, we have

$$\begin{aligned} 2500q &= q^2 + 1200q + (600)^2 \\ 0 &= q^2 - 1300q + 360{,}000 \end{aligned}$$

By the quadratic formula,

$$\begin{aligned} q &= \frac{1300 \pm \sqrt{250{,}000}}{2} \\ q &= \frac{1300 \pm 500}{2} \\ q &= 400 \quad \text{or} \quad q = 900 \end{aligned}$$

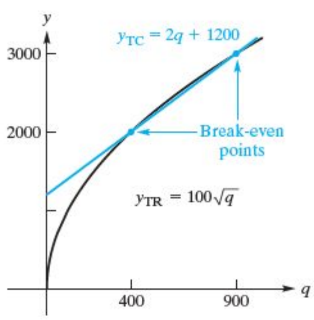

FIGURE 3.49 Two break-even points.

Although both $q = 400$ and $q = 900$ are break-even quantities, observe in Figure 3.49 that when $q > 900$, total cost is greater than total revenue, so there will always be a loss. This occurs because here total revenue is not linearly related to output. Thus, producing more than the break-even quantity does not necessarily guarantee a profit.

Now Work Problem 21 ◁

PROBLEMS 3.6

In Problems 1–8, you are given a supply equation and a demand equation for a product. If p represents price per unit in dollars and q represents the number of units per unit of time, find the equilibrium point. In Problems 1 and 2, sketch the system.

1. Supply: $p = \frac{2}{100}q + 3$, Demand: $p = -\frac{3}{100}q + 11$

2. Supply: $p = \frac{1}{1500}q + 4$, Demand: $p = -\frac{1}{2000}q + 9$

3. Supply: $35q - 2p + 250 = 0$, Demand: $65q + p - 537.5 = 0$

4. Supply: $246p - 3.25q - 2460 = 0$, Demand: $410p + 3q - 14{,}452.5 = 0$

5. Supply: $p = 2q + 20$, Demand: $p = 200 - 2q^2$

6. Supply: $p = (q + 12)^2$, Demand: $p = 644 - 6q - q^2$

7. Supply: $p = \sqrt{q + 10}$, Demand: $p = 20 - q$

8. Supply: $p = \frac{1}{4}q + 6$, Demand: $p = \frac{2240}{q + 12}$

In Problems 9–14, y_{TR} represents total revenue in dollars and y_{TC} represents total cost in dollars for a manufacturer. If q represents both the number of units produced and the number of units sold, find the break-even quantity. Sketch a break-even chart in Problems 9 and 10.

9. $y_{TR} = 4q$; $y_{TC} = 2q + 5000$

10. $y_{TR} = 14q$; $y_{TC} = \frac{40}{3}q + 1200$

11. $y_{TR} = 0.02q$; $y_{TC} = 0.5q + 30$

12. $y_{TR} = 0.25q$; $y_{TC} = 0.16q + 360$

13. $y_{TR} = 90 - \dfrac{900}{q+3}$; $y_{TC} = 1.1q + 37.3$

14. $y_{TR} = 0.1q^2 + 9q$; $y_{TC} = 3q + 400$

15. Business Supply and demand equations for a certain product are

$$3q - 200p + 1800 = 0$$

and

$$3q + 100p - 1800 = 0$$

respectively, where p represents the price per unit in dollars and q represents the number of units sold per time period.
(a) Find the equilibrium price algebraically, and derive it graphically.
(b) Find the equilibrium price when a tax of 27 cents per unit is imposed on the supplier.

16. Business A manufacturer of a product sells all that is produced. The total revenue is given by $y_{TR} = 8q$, and the total cost is given by $y_{TC} = 7q + 500$, where q represents the number of units produced and sold.
(a) Find the level of production at the break-even point, and draw the break-even chart.
(b) Find the level of production at the break-even point if the total cost increases by 4%.

17. Business A manufacturer sells a product at \$8.35 per unit, selling all produced. The fixed cost is \$2116 and the variable cost is \$7.20 per unit. At what level of production will there be a profit of \$4600? At what level of production will there be a loss of \$1150? At what level of production will the break-even point occur?

18. Business The market equilibrium point for a product occurs when 13,500 units are produced at a price of \$4.50 per unit. The producer will supply no units at \$1, and the consumers will demand no units at \$20. Find the supply and demand equations if they are both linear.

19. Business A manufacturer of a children's toy will break even at a sales volume of \$200,000. Fixed costs are \$40,000, and each unit of output sells for \$5. Determine the variable cost per unit.

20. Business The Bigfoot Sandal Co. manufactures sandals for which the material cost is \$0.85 per pair and the labor cost is \$0.96 per pair. Additional variable costs amount to \$0.32 per pair. Fixed costs are \$70,500. If each pair sells for \$2.63, how many pairs must be sold for the company to break even?

21. Business **(a)** Find the break-even points for company X, which sells all it produces, if the variable cost per unit is \$3, fixed costs are \$2, and $y_{TR} = 5\sqrt{q}$, where q is the number of thousands of units of output produced.
(b) Graph the total revenue curve and the total cost curve in the same plane.
(c) Use your answer in **(a)** to report the quantity interval in which maximum profit occurs.

22. Business A company has determined that the demand equation for its product is $p = 1000/q$, where p is the price per unit for q units produced and sold in some period. Determine the quantity demanded when the price per unit is (a) \$4, (b) \$2, and (c) \$0.50. For each of these prices, determine the total revenue that the company will receive. What will be the revenue regardless of the price? [*Hint:* Find the revenue when the price is p dollars.]

23. Business Using the data in Example 1, determine how the original equilibrium price will be affected if the company is given a government subsidy of \$1.50 per unit.

24. Business The Monroe Forging Company sells a corrugated steel product to the Standard Manufacturing Company and is in competition on such sales with other suppliers of the Standard Manufacturing Co. The vice president of sales of Monroe Forging Co. believes that by reducing the price of the product, a 40% increase in the volume of units sold to the Standard Manufacturing Co. could be secured. As the manager of the cost and analysis department, you have been asked to analyze the proposal of the vice president and submit your recommendations as to whether it is financially beneficial to the Monroe Forging Co. You are specifically requested to determine the following:
(a) Net profit or loss based on the pricing proposal
(b) Unit sales volume under the proposed price that is required to make the same \$40,000 profit that is now earned at the current price and unit sales volume
Use the following data in your analysis:

	Current Operations	Proposal of Vice President of Sales
Unit price	\$2.50	\$2.00
Unit sales volume	200,000 units	280,000 units
Variable cost		
Total	\$350,000	\$490,000
Per unit	\$1.75	\$1.75
Fixed cost	\$110,000	\$110,000
Profit	\$40,000	?

25. Business Suppose products A and B have demand and supply equations that are related to each other. If q_A and q_B are the quantities produced and sold of A and B, respectively, and p_A and p_B are their respective prices, the demand equations are

$$q_A = 7 - p_A + p_B$$

and

$$q_B = 24 + p_A - p_B$$

and the supply equations are

$$q_A = -3 + 4p_A - 2p_B$$

and

$$q_B = -5 - 2p_A + 4p_B$$

Eliminate q_A and q_B to get the equilibrium prices.