Section 3.4   Systems of Linear Equations   **157**

solution of the given system is

$$x = 4 - 2r - s$$
$$y = r$$
$$z = s$$

where r and s can be any real numbers. Each assignment of values to r and s results in a solution of the given system, so there are infinitely many solutions. For example, letting $r = 1$ and $s = 2$ gives the particular solution $x = 0, y = 1$, and $z = 2$. As in the last example, there is nothing special about the names of the parameters. In particular, since $y = r$ and $z = s$, we could consider y and z to be the two parameters.

Now Work Problem 23 ◁

PROBLEMS 3.4

In Problems 1–24, solve the systems algebraically.

1. $\begin{cases} x + 4y = 3 \\ 3x - 2y = -5 \end{cases}$ 2. $\begin{cases} 4x + 2y = 9 \\ 5y - 4x = 5 \end{cases}$

3. $\begin{cases} 2x + 3y = 1 \\ x + 2y = 0 \end{cases}$ 4. $\begin{cases} 2x - y = 1 \\ -x + 2y = 7 \end{cases}$

5. $\begin{cases} u + v = 5 \\ u - v = 7 \end{cases}$ 6. $\begin{cases} 2p + q = 16 \\ 3p + 3q = 33 \end{cases}$

7. $\begin{cases} x - 2y = -7 \\ 5x + 3y = -9 \end{cases}$ 8. $\begin{cases} 4x + 12y = 12 \\ 2x + 4y = 12 \end{cases}$

9. $\begin{cases} 4x - 3y - 2 = 3x - 7y \\ x + 5y - 2 = y + 4 \end{cases}$

10. $\begin{cases} 5x + 7y + 2 = 9y - 4x + 6 \\ \frac{21}{2}x - \frac{4}{3}y - \frac{11}{4} = \frac{3}{2}x + \frac{2}{3}y + \frac{5}{4} \end{cases}$

11. $\begin{cases} \frac{2}{3}x + \frac{1}{2}y = 2 \\ \frac{3}{8}x + \frac{5}{6}y = -\frac{11}{2} \end{cases}$ 12. $\begin{cases} \frac{1}{2}z - \frac{1}{4}w = \frac{1}{6} \\ \frac{1}{2}z + \frac{1}{4}w = \frac{1}{6} \end{cases}$

13. $\begin{cases} 2p + 3q = 5 \\ 10p + 15q = 25 \end{cases}$ 14. $\begin{cases} 5x - 3y = 2 \\ -10x + 6y = 4 \end{cases}$

15. $\begin{cases} 2x + y + 6z = 3 \\ x - y + 4z = 1 \\ 3x + 2y - 2z = 2 \end{cases}$ 16. $\begin{cases} x + y + z = -1 \\ 3x + y + z = 1 \\ 4x - 2y + 2z = 0 \end{cases}$

17. $\begin{cases} x + 4y + 3z = 10 \\ 4x + 2y - 2z = -2 \\ 3x - y + z = 11 \end{cases}$ 18. $\begin{cases} x + 2y + z = 4 \\ 2x - 4y - 5z = 26 \\ 2x + 3y + z = 10 \end{cases}$

19. $\begin{cases} x - 2z = 1 \\ y + z = 3 \end{cases}$ 20. $\begin{cases} 2y + 3z = 1 \\ 3x - 4z = 0 \end{cases}$

21. $\begin{cases} x - y + 2z = 0 \\ 2x + y - z = 0 \\ x + 2y - 3z = 0 \end{cases}$ 22. $\begin{cases} x - 2y - z = 0 \\ 2x - 4y - 2z = 0 \\ -x + 2y + z = 0 \end{cases}$

23. $\begin{cases} x - 3y + z = 5 \\ -2x + 6y - 2z = -10 \end{cases}$ 24. $\begin{cases} 5x + y + z = 17 \\ 4x + y + z = 14 \end{cases}$

25. Mixture   A chemical manufacturer wishes to fill an order for 800 gallons of a 25% acid solution. Solutions of 20% and 35% are in stock. How many gallons of each solution must be mixed to fill the order?

26. Mixture   A gardener has two fertilizers that contain different concentrations of nitrogen. One is 3% nitrogen and the other is 11% nitrogen. How many pounds of each should she mix to obtain 20 pounds of a 9% concentration?

27. Fabric   A textile mill produces fabric made from different fibers. From cotton, polyester, and nylon, the owners want to produce a fabric blend that will cost $3.25 per pound to make. The cost per pound of these fibers is $4.00, $3.00, and $2.00, respectively. The amount of nylon is to be the same as the amount of polyester. How much of each fiber will be in the final fabric?

28. Taxes   A company has taxable income of $758,000. The federal tax is 35% of that portion left after the state tax has been paid. The state tax is 15% of that portion left after the federal tax has been paid. Find the federal and state taxes.

29. Airplane Speed   An airplane travels 900 mi in 2 h, 55 min, with the aid of a tailwind. It takes 3 h, 26 min, for the return trip, flying against the same wind. Find the speed of the airplane in still air and the speed of the wind.
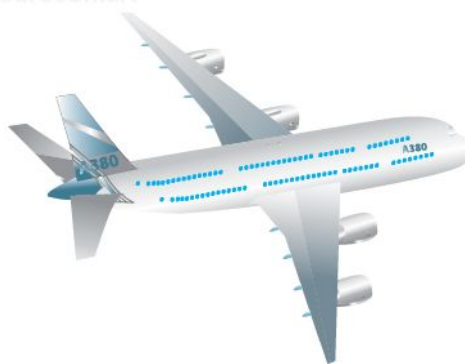
30. Speed of Raft   On a trip on a raft, it took $\frac{1}{2}$ hour to travel 10 miles downstream. The return trip took $\frac{3}{4}$ hour. Find the speed of the raft in still water and the speed of the current.

31. Furniture Sales   A manufacturer of dining-room sets produces two styles: early American and contemporary. From past experience, management has determined that 20% more of the early American styles can be sold than the contemporary styles. A profit of $250 is made on each early American set sold, whereas a profit of $350 is made on each contemporary set. If, in the forthcoming year, management desires a total profit of $130,000, how many units of each style must be sold?

32. Survey   National Surveys was awarded a contract to perform a product-rating survey for Crispy Crackers. A total of