We have

$$\begin{aligned} r &= pq \\ &= (1000 - 2q)q \\ r &= 1000q - 2q^2 \end{aligned}$$

Note that r is a quadratic function of q, with $a = -2, b = 1000$, and $c = 0$. Since $a < 0$ (the parabola opens downward), r is maximum at the vertex (q, r), where

$$q = -\frac{b}{2a} = -\frac{1000}{2(-2)} = 250$$

The maximum value of r is given by

$$\begin{aligned} r &= 1000(250) - 2(250)^2 \\ &= 250{,}000 - 125{,}000 = 125{,}000 \end{aligned}$$

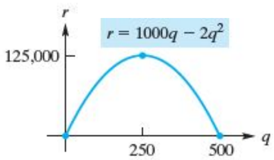

FIGURE 3.25 Graph of revenue function.

Thus, the maximum revenue that the manufacturer can receive is \$125,000, which occurs at a production level of 250 units. Figure 3.25 shows the graph of the revenue function. Only that portion for which $q \geq 0$ and $r \geq 0$ is drawn, since quantity and revenue cannot be negative.

Now Work Problem 29 ◁

PROBLEMS 3.3

In Problems 1–8, state whether the function is quadratic.

1. $f(x) = 5x^2$ **2.** $g(x) = \frac{1}{2x^2 - 4}$

3. $g(x) = 7 - 6x$ **4.** $k(v) = 2v^2(2v^2 + 2)$

5. $h(q) = (3 - q)^2$ **6.** $f(t) = 2t(3 - t) + 4t$

7. $f(s) = \frac{s^2 - 9}{2}$ **8.** $g(t) = (t^2 - 1)^2$

In Problems 9–12, do not include a graph.

9. (a) For the parabola $y = f(x) = 3x^2 + 5x + 1$, find the vertex. (b) Does the vertex correspond to the highest point or the lowest point on the graph?

10. Repeat Problem 9 if $y = f(x) = 8x^2 + 4x - 1$.

11. For the parabola $y = f(x) = x^2 + x - 6$, find (a) the y-intercept, (b) the x-intercepts, and (c) the vertex.

12. Repeat Problem 11 if $y = f(x) = 5 - x - 3x^2$.

In Problems 13–22, graph each function. Give the vertex and intercepts, and state the range.

13. $y = f(x) = x^2 - 6x + 5$ **14.** $y = f(x) = 9x^2$

15. $y = g(x) = -2x^2 - 6x$ **16.** $y = f(x) = x^2 - 4$

17. $s = h(t) = t^2 + 6t + 9$ **18.** $s = h(t) = 2t^2 + 3t - 2$

19. $y = f(x) = -5 + 3x - 3x^2$

20. $y = H(x) = 1 - x - x^2$

21. $t = f(s) = s^2 - 8s + 14$

22. $t = f(s) = s^2 + 6s + 11$

In Problems 23–26, state whether $f(x)$ has a maximum value or a minimum value, and find that value.

23. $f(x) = 49x^2 - 10x + 17$ **24.** $f(x) = -7x^2 - 2x + 6$

25. $f(x) = 4x - 50 - 0.1x^2$ **26.** $f(x) = x(x + 3) - 12$

In Problems 27 and 28, restrict the quadratic function to those x satisfying $x \geq v$, where v is the x-coordinate of the vertex of the parabola. Determine the inverse of the restricted function. Graph the restricted function and its inverse in the same plane.

27. $f(x) = x^2 - 2x + 4$ **28.** $f(x) = -x^2 + 4x - 3$

29. Revenue The demand function for a manufacturer's product is $p = f(q) = 100 - 10q$, where p is the price (in dollars) per unit when q units are demanded (per day). Find the level of production that maximizes the manufacturer's total revenue and determine this revenue.

30. Revenue The demand function for an office supply company's line of plastic rulers is $p = 0.85 - 0.00045q$, where p is the price (in dollars) per unit when q units are demanded (per day) by consumers. Find the level of production that will maximize the manufacturer's total revenue, and determine this revenue.

31. Revenue The demand function for an electronics company's laptop computer line is $p = 2400 - 6q$, where p is the price (in dollars) per unit when q units are demanded (per week) by consumers. Find the level of production that will maximize the manufacturer's total revenue, and determine this revenue.

32. Marketing A marketing firm estimates that n months after the introduction of a client's new product, $f(n)$ thousand households will use it, where

$$f(n) = \frac{10}{9}n(12 - n), \quad 0 \leq n \leq 12$$

Estimate the maximum number of households that will use the product.

33. Profit The daily profit for the garden department of a store from the sale of trees is given by $P(x) = -x^2 + 18x + 144$, where x is the number of trees sold. Find the function's vertex and intercepts, and graph the function.

34. Psychology A prediction made by early psychology relating the magnitude of a stimulus, x, to the magnitude of a response, y, is expressed by the equation $y = kx^2$, where k is