

Panel 1

Summary

Compound interest: $S = P(1+r)^n$

Effective rate: $(1 + \frac{r}{n})^n - 1 = r_e$

Present Value: $= PV(\text{rate}, \text{uper}, \text{pmt}, \text{fv})$

Future Value: $= FV(\text{rate}, \text{uper}, \text{pmt}, \text{pv})$

Payments: $= PMT(\text{rate}, \text{uper}, \text{pv}, 0)$

(new)

continuous comp: $S = P e^{rt}$

negative

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Panel 2

- ① You sign for a loan with payments of \$200 per month at 10% over 10 years compounded monthly
 - ② You want to setup a trust fund of \$50,000 in 15 years at 3% interest compounded monthly with monthly payments of \$150,-
 - ③ You invest \$150 per month at 7% compounded monthly for 4 years. How much will you have?
 - ④ You take out a loan for \$20,000 over 5 years at 4% interest compounded monthly. Payments per month?
- 2

Panel 3

① You sign for a loan with payments of \$200 per month at 10% over 10 years compounded monthly. How much money do you get? (now)

$$= PV\left(\frac{0.1}{12}, 10 \times 12, -200, 0\right) = \underline{\$15,143.23}$$

less than:

$$\text{I pay } \$200 \times 12 \times 10 = \underline{\$24,000}$$

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Panel 4

② You want to setup a trust fund of \$50,000 in 15 years at 3% interest compounded monthly with monthly payments of \$150. What's the principle to pay initially?

$$= PV\left(\frac{0.03}{12}, 15 \times 12, -150, 50000\right) = \underline{\$10,125.50}$$

$$\text{You pay } 15 \times 12 \times \$150 = \$27,000 - \begin{matrix} \uparrow \\ \text{less than } \$23k \end{matrix}$$

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Panel 5

③ You invest \$150 per month at 7% compounded monthly for 4 years. How much will you have?

$$= FV\left(\frac{0.07}{12}, 4 \cdot 12, -150, 0\right) = \underline{\underline{\$8291.39}}$$

Estimate: HW

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Panel 6

④ ^{New} You take out a loan for \$20,000 over 5 years at 4% interest compounded monthly. Payments per month?

$$= PMT\left(\frac{0.04}{12}, 12 \cdot 5, -20000, 0\right) = \underline{\underline{\$368.73}}$$

I end up paying: $60 \cdot 368.73 = \underline{\underline{\$22092.30}}$

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Panel 7

Last question: Suppose you want to purchase a house.
 You need a loan of \$200,000 over 30 years.
 Bank A offers 5.6%, Bank B offers 3.625%
 What is the savings over the life of the loan if
 you make monthly payments and monthly compounding.

$$= \text{PMT} \left(\frac{0.056}{12}, 30 \cdot 12, -200000, 0 \right) = \underline{\$1149.10}$$

$$= \text{PMT} \left(\frac{0.03625}{12}, 30 \cdot 12, -200000, 0 \right) = \$914.92$$

Over 30 year Bank A: \$413,336
 Bank B: \$329,326
) difference is
 \$84,000

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Panel 8

Done will finance math!

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Panel 9

Last Topic: Solving Systems of Equations

Want to solve

$$3x - y = 1$$

$$x + 2y = 5$$

subst.elimination

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Panel 10

Matrix Method

$$3x - y = 1$$

$$1x + 2y = 5$$

$$\begin{matrix} x & y \\ \left(\begin{array}{cc} 3 & -1 \\ 1 & 2 \end{array} \right) \end{matrix}$$

Definition: (Coefficient Matrix)

The coefficients of linear equations arranged in a rectangular grid.

$$\begin{matrix} x & y & = \\ \left(\begin{array}{ccc} 3 & -1 & 1 \\ 1 & 2 & 5 \end{array} \right) \end{matrix}$$

Def: (Augmented coefficient matrix)
coefficient matrix + right side

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Panel 11

Ex: Find the augmented coefficient matrix for the system:

$$\begin{array}{r} 2x + y - z = 3 \\ -2x + 2y + 3z = 8 \\ x + y + z = 1 \end{array} \quad \begin{array}{c} x \quad y \quad z \quad = \\ \left(\begin{array}{ccc|c} 2 & 1 & -1 & 3 \\ -2 & 2 & 3 & 8 \\ 1 & 1 & 1 & 1 \end{array} \right) \end{array}$$

Ex: Find the augmented coefficient matrix for:

$$\begin{array}{r} x + 2y + 3z = 11 \\ y + 2z = 5 \\ 3z = 6 \end{array} \quad \left(\begin{array}{ccc|c} 1 & 2 & 3 & 11 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 3 & 6 \end{array} \right)$$

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Panel 12

Ex: If the augmented matrix of a system of equations is:

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 5 \end{array} \right) \quad \begin{array}{l} x=4 \\ y=2 \\ z=5 \end{array}$$

what is the solution for the original system?

Ex: Solve a system of equations with augmented matrix

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 3 & 7 \\ 0 & 0 & 1 & 2 \end{array} \right) \quad \begin{array}{l} x + z = 2 \Rightarrow \underline{x=0} \\ y + 3z = 7, y + 6 = 7, \underline{y=1} \\ \underline{z=2} \end{array}$$

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Panel 13

So: If an augmented matrix is:

$$\begin{pmatrix} 2 & 1 & -1 & 3 \\ -2 & 2 & 3 & 8 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

can not find the answer

But Goal if matrix was

$$\underline{\underline{x=3}}$$

$$\begin{pmatrix} 1 & 2 & 3 & 11 \\ 0 & 1 & 2 & 7 \\ 0 & 0 & 3 & 6 \end{pmatrix}$$

$$x + 2y + 3z = 11 \rightarrow x + 2 + 6 = 11$$

$$y + 2z = 7 \rightarrow y + 4 = 7 \underline{\underline{y=3}}$$

$$3z = 6 \rightarrow \underline{\underline{z=2}}$$

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Panel 14

① Eliminate the variable circled from the system:

$$\left. \begin{array}{l} 2x + y - z = 3 \\ -2x + 2y + 3z = 8 \\ x + y + z = 1 \end{array} \right\} \Rightarrow \begin{array}{l} 2x + y - z = 3 \\ 0 + 3y + 2z = 11 \\ x + y + z = 1 \end{array}$$

$$\left(\begin{array}{cccc} 2 & 1 & -1 & 3 \\ -2 & 2 & 3 & 8 \\ 1 & 1 & 1 & 1 \end{array} \right) \xrightarrow{+2_3} \left(\begin{array}{cccc} 2 & 1 & -1 & 3 \\ 0 & 3 & 2 & 11 \\ 1 & 1 & 1 & 1 \end{array} \right)$$

Good!

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Panel 15

Elementary Matrix Operations

You are allowed to perform the following operations on an augmented matrix without changing the solution:

- (1) Swap 2 rows
- (2) Mult. row by any # ($\neq 0$)
- (3) Add two rows and replace one of them with sum.

Ex: $2x + 4y = 8$
 $-x + y = 2$

$$\rightarrow \left(\begin{array}{cc|c} 2 & 4 & 8 \\ -1 & 1 & 2 \end{array} \right) \rightarrow \left(\begin{array}{cc|c} 2 & 4 & 8 \\ -2 & 2 & 4 \end{array} \right) \begin{array}{l} x=0 \\ x+y=1 \\ 2x+4y=8 \end{array}$$

$$\rightarrow \left(\begin{array}{cc|c} 2 & 4 & 8 \\ 0 & 6 & 12 \end{array} \right) \begin{array}{l} x=0 \\ y=2 \end{array}$$

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Solving a System by Elementary Operations

Solve $2x + y = 3$

$x + y + 3z = 1$

$x + 2y + 4z = 6$

$$\left(\begin{array}{ccc|c} 0 & 1 & 2 & 3 \\ 1 & 1 & 3 & 1 \\ 1 & 2 & 4 & 6 \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} -1 & -1 & -3 & -1 \\ 0 & 1 & 2 & 3 \\ 1 & 2 & 4 & 6 \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} -1 & -1 & -3 & -1 \\ 0 & -1 & -2 & -3 \\ 0 & 1 & 1 & 7 \end{array} \right)$$

Goal: make these zero!

$x=0$

$x + y + 3z = 1$
 $x + 2z = 6$

$$\left(\begin{array}{ccc|c} 1 & 1 & 3 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & -2 \end{array} \right) \begin{array}{l} y - 4 = 3, y = 7 \\ y + 2z = 3 \\ z = -2 \end{array}$$

Panel 17

- Legal operations:
- ① Multiply any row by any number
 - ② Replace any row by sum/difference of rows
 - ③ Switch any two rows

Gauss-Jordan Elimination Method

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Panel 18

Solve $2x + y + z = 3$

$$-x + 2y + 2z = 1$$

$$x - y - 3z = -6$$

$$\begin{pmatrix} 2 & 1 & 1 & 3 \\ -1 & 2 & 2 & 1 \\ 1 & -1 & -3 & -6 \end{pmatrix} \xrightarrow{\substack{\downarrow \\ \downarrow \\ \downarrow}} \begin{pmatrix} 2 & 1 & 1 & 3 \\ 0 & 1 & -1 & -5 \\ 1 & -1 & -3 & -6 \end{pmatrix} \xrightarrow{\text{swap}} \begin{pmatrix} 2 & 1 & 1 & 3 \\ 0 & 1 & -1 & -5 \\ -2 & 2 & 6 & 12 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 & 1 & 3 \\ 0 & 1 & -1 & -5 \\ 0 & 3 & 7 & 17 \end{pmatrix} \xrightarrow{\text{R}_3 - 3\text{R}_2} \begin{pmatrix} 2 & 1 & 1 & 3 \\ 0 & 1 & -1 & -5 \\ 0 & 3 & 7 & 17 \end{pmatrix} \xrightarrow{\text{R}_3 - 3\text{R}_2} \begin{pmatrix} 2 & 1 & 1 & 3 \\ 0 & 1 & -1 & -5 \\ 0 & 0 & 10 & 30 \end{pmatrix}$$

$$10z = 30, \quad z = 3, \quad -3y + 3z = 17, \quad -3y + 9 = 17, \quad -3y = 6, \quad y = -2$$

$$2x + y + z = 3 \Rightarrow 2x - 2 + 3 = 3, \quad 2x = 2, \quad x = 1$$

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Panel 19

Ex: Solve the following system of linear equations:

$$x + y + z = 0$$

$$-x + 2y + z = 3$$

$$-x - 4y - 2z = 0$$

Hw

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Panel 20

Ex: $2x + y + 6z = 3$

$$x - y + 4z = 1$$

$$3x + 2y - 2z = 2$$

Hw

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Panel 21

$$\underline{\text{Ex:}} \quad \begin{aligned} 5p + 11q &= 7 \\ 10p + 22q &= 33 \end{aligned}$$

HW

$$\underline{\text{Ex:}} \quad \begin{aligned} x - y + 2z &= 0 \\ 2x + y - z &= 0 \\ x + 2y - 3z &= 0 \end{aligned}$$

HW

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Panel 22

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