Panel 1

Exam 3: Nov 30^{th}

Final: Dec 14^{th} @ 7:45 am

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Panel 2

Last Time.

Area under curve: $\int_a^b f(x)\,dx$ if $f(x) \geq 0$

Area between curves: $\int_a^b f(x) - g(x)\,dx$ if $f(x) \geq g(x)$

Recall Fund. Thm. of Calc: $\int_a^b f(x)\,dx = F(b) - F(a)$, F antideriv.

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Panel 3

Quiz #9 Name: ______

(1) Find area under curve $f(x) = 9 - x^2$ from $x = -3$ to $x = 3$.

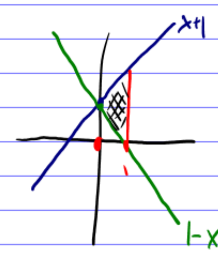
$$\int_{-3}^{3} 9 - x^2\,dx = 9x - \frac{1}{3}x^3 \Big|_{-3}^{3} = \left(9\cdot 3 - \frac{1}{3}3^3\right) - \left(-27 + 9\right.$$

$$= 27 + 27 - 9 - 9 = 36$$

(2) Find area between $f(x) = x + 1$, $g(x) = 1 - x$, as $x = 0$ to $x = 1$

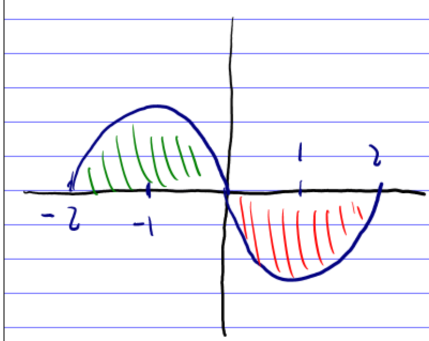
(1) $\int_0^1 (x+1) - (1-x)\,dx = \int_0^1 2x\,dx = 2\cdot\frac{1}{2}x^2\Big|_0^1 = x^2\Big|_0^1 = 1 - 0 = 1$

(2) or $\int_0^1 (1-x) - (x+1)\,dx = -1$



Panel 4

(3) Consider the function $f(x)$ whose graph is shown below.
Find the signs (pos, neg, zero) of:



a) $f'(0)$ < 0

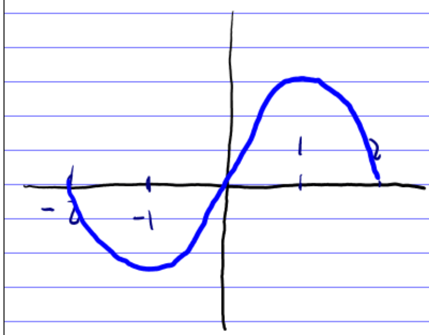
b) $f''(1)$ > 0

c) $\int_{-2}^{0} f(x)\,dx$ > 0

d) $\int_{-2}^{2} f(x)\,dx$ zero

Panel 5

(3) Consider the function $f(x)$ whose graph is shown below.
Find the signs (pos., neg., zero) of:



a) $f'(0)$

b) $f''(1)$

c) $\int_{-2}^{0} f(x)\,dx$

$\int_{-2}^{1} f(x)\,dx$

d) $\int_{-2}^{2} f(x)\,dx$

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Panel 6

<u>Final Topics:</u> Financial Mathematics

Recall <u>Compound Interest Formula:</u> If you invest a principal P at an interest rate r per period compounded for n periods in total, you have:

$$S = P(1+r)^n$$

<u>Ex:</u> \$1000 at 8% compounded quarterly for 5 years:

$$S = 1000\left(1+\frac{0.08}{4}\right)^{20} = \$1485.95$$

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Panel 7

If you invest \$P at 10% compounded quarterly for one year, it will earn more than 10% that year.

$S - P = P\left(1+\frac{0.1}{4}\right)^4 - P = P\left[1.025^4 - 1\right] = P(1.1038 - 1) = P \cdot 0.1038$

So 10.38% of P

Def: The effective rate r_e equivalent to a nominal rate r compounded n times per year is:

$r_e = \left(1+\frac{r}{n}\right)^n - 1$

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Panel 8

Effective rate $r_e = \left(1 + r/n\right)^n - 1$

Ex: Find effective rate equivalent to 6% compounded (a) semiannually, (b) monthly

a) $\left(1+\frac{0.06}{2}\right)^2 - 1 = 0.0609$ or 6.09%

b) $\left(1+\frac{0.06}{12}\right)^{12} - 1 = 0.0617$ or 6.17%

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Panel 9

Effective rate per year = nominal rate per period, compounded n-times.

Ex: Nominal rate of 5% over 10 years compounded monthly. Start with \$2000

$$2000\left(1+\frac{0.05}{12}\right)^{120} = \$3294$$

$$r_e = \left(1+\frac{0.05}{12}\right)^{12} - 1 = 0.05116 \text{ or } 5.116\%$$

Same

$$2000(1+0.05116)^{10} = \$3293.95$$

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Panel 10

Present Value

Usually want to know how much money we have after X years.

Sometimes I want the opposite: know that I will need \$X in the future (say in 10 years), how much should I invest now to get \$X in the future.

Present Value: the amount P to invest now to have X in the future.

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Panel 11

Ex: Suppose you need \$1,000 in three years, and your bank offers 9% compounded monthly. Find the present value of \$1,000 in 3 years.

Want Present value of \$1000 in 3 years.

$S = P(1+r)^n$, $1000 = P\left(1+\frac{0.09}{12}\right)^{36}$

$\Rightarrow 1000\left(1+\frac{0.09}{12}\right)^{-36} = P$

\$764.15 = P

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Panel 12

Present Value	*The PV function returns the present value of an investment. The present value is the total amount that a series of future payments is worth now. For example, when you borrow money, the loan amount is the present value to the lender.*	
Excel Formula:	**=PV(rate, nper, pmt, fv)**, where	
	rate:	is the interest rate per period.
	nper:	is the total number of payment periods in an annuity.
	pmt:	is the payment made each period; it cannot change over the life of the annuity. (negative)
	fv:	is the future value, or a cash balance you want to attain after the last payment is made. (positive)

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Panel 13

Example: *Find the present value of $1000 due after 3 years if the interest rate is 9% compounded monthly.*

Solution:			
	Interest rate:	*0.09*	
	Compound Periods:	*12*	
	Number of years:	*3*	
	rate:	**0.0075**	= *Interest rate / compound periods*
	nper:	**36**	= *Number of years * compound periods*
	pmt:	**0**	
	fv:	**1000**	
	PV(rate, nper, pmt, fv) =	**($764.15)**	

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Panel 14

Ex: A trust fund for a child should yield $50,000 in 15 years at 7% comp. semiannually. How much should you invest?

=PV(0.07/2,15*2,0,50000) = $17,813.92

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Panel 15

Ex: Trust fund should yield $50.000 after 15 years, at 7% compounded semiannually, and every period we make a payment of $100.-

=PV(0.07/2,15*2,-100,50000) = $15979

neg

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Panel 16

Other useful Excel formulas for financial math

Future value:

=FV(rate, nper, pmt, pv)

rate = rate per period

nper = # of periods

pmt = constant payment per period (negative)

pv = present value (principle) (negative)

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Panel 17

Ex: Invest \$1000 at 8% compounded quarterly for 5 years.

=

=FV(0.08/4,5*4,0,-1000) = \$1485.95

$S = \$1000\left(1+\frac{0.08}{4}\right)^{20} =$ = \$1485.95

does not include monthly payments

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Panel 18

Ex: Value of \$1000 at 8% compounded quarterly in 5 years if \$100 payments are made every quarter.

=FV(0.08/4, 4*5,-100,-1000) = \$3915.16

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Panel 19

Ex: What is the present value of \$3915.68 in 5 years at 8% compounded quarterly, if \$100 payments are made?

$$= PV(0.08/4, 5\cdot 4, -100, 3915.68) = \$1000$$, obviously (see prev. problem)

$\Rightarrow$ PV and FV are inverse of each other

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