Panel 1

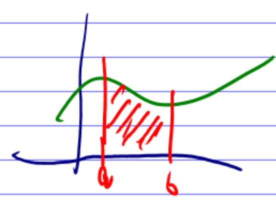
Last Time

$\int f(x)\,dx$ = antiderivative $+C$ (indefinite integral)

integral of $f(x)$

$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$ (definite integral)

represents graphically area under $f(x)$ as long as $f(x) \geq 0$.



1

Panel 2

Ex: $\int x^2 + \sqrt{x^3} + 1/x \, dx = \frac{1}{3}x^3 + \frac{2}{5}x^{5/2} + \ln|x| + C$

$\sqrt{x^3} = x^{3/2}$

$\int_0^1 4x - 9x^2\,dx = 4\frac{1}{2}x^2 - 9\frac{1}{3}x^3\Big|_0^1 = 2x^2 - 3x^3\Big|_0^1 = (2-3)-(0-0) = -1$

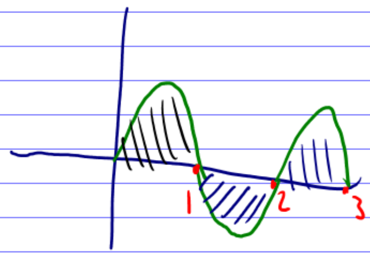
Ex: Area under $f(x) = 3x^2+1$ from $x=1$ to 2

$A = \int_1^2 3x^2+1\,dx = x^3 + x\Big|_1^2 = (2^3+2) - (1+1) = 8$

is positive

2

Panel 3



$\int_0^1 f(x)\,dx$ pos

$\int_0^2 f(x)\,dx$ zero

$\int_0^3 f(x)\,dx$ pos

$\int_1^2 f(x)\,dx$ neg.

$\int_1^3 f(x)\,dx$ zero

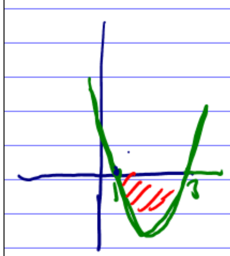
3

Panel 4

Ex: Area under curve $f(x) = x^2 - 4x + 3$

Q: What are the bounds for x?

$$\int x^2-4x+3\,dx = \int_1^3 x^2-4x+3\,dx = \frac{1}{3}x^3 - 2x^2 + 3x\Big|_1^3 =$$



$$= (9 - 18 + 9) - \left(\frac{1}{3} - 2 + 3\right) =$$

$$= -4/3$$

is the value of integral.

$x^2 - 4x + 3 = (x-3)(x-1)$

But: area is going to be $4/3$

Watch out for negatives (and fractions)

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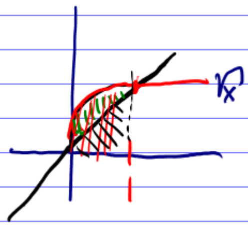
Panel 5

Area between Curves

Find area between $y = x$ and $y = \sqrt{x}$ from $x = 0$ to $x = 1$



A = red − black =

$$\int_0^1 \sqrt{x}\,dx - \int_0^1 x\,dx =$$

$$= \int_0^1 \sqrt{x} - x\,dx = \frac{2}{3}x^{3/2} - \frac{1}{2}x^2 \Big|_0^1 = \left(\frac{2}{3} - \frac{1}{2}\right) - (0 - 0) =$$

$$= \frac{4}{6} - \frac{3}{6} = \frac{1}{6}$$

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Panel 6

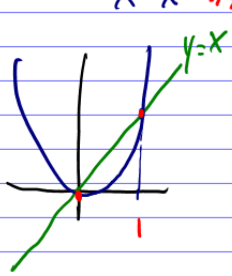
Def: Area between $f(x)$ and $g(x)$ is $\int_a^b f(x) - g(x)\,dx$, as long as $f - g \geq 0$ or $f \geq g$

Ex: Area between $f(x) = x^2$ and $g(x) = x$ from $x = 0$ to $x = 1$

(1) $\int_0^1 x^2 - x\,dx$ (2) or $\int_0^1 x - x^2\,dx$

$X = x^2 \Rightarrow x = 0, 1$

$$\int_0^1 x^2 - x\,dx = \frac{1}{3}x^3 - \frac{1}{2}x^2 \Big|_0^1 = \left(\frac{1}{3} - \frac{1}{2}\right) - (0) = \frac{1}{3} - \frac{1}{2} = -\frac{1}{6}$$



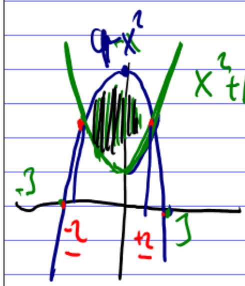
can't be negative so $\int_0^1 x - x^2\,dx = \frac{1}{6}$

6

Panel 7

Ex: Area between $9-x^2$ and x^2+1 (what are bounds)

① $A = \int_{?}^{?} (9-x^2)-(x^2+1)\,dx$ or ~~② $\int_{?}^{?} (x^2+1)-(9-x^2)\,dx$~~



$9-x^2 = x^2+1$

$8 = 2x^2$

$4 = x^2$

$\pm 2 = x$

Want bigger smaller

$A = \int_{-2}^{2} (9-x^2)-(x^2+1)\,dx =$

$= \int_{-2}^{2} 8-2x^2\,dx = 8x - \frac{2}{3}x^3 \Big|_{-2}^{2} =$

$= \left(8\cdot 2 - \frac{2}{3}2^3\right) - \left(8(-2) - \frac{2}{3}(-2)^3\right)$

$16 - \frac{16}{3} + 16 - \frac{16}{3} = 32 - \frac{32}{3} = \frac{64}{3}$

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Panel 8

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6. Find the area under the graph $y = e^x + 2$ from x = -2 to x = 1. Sketch and shade the region. (6%)

e^x+2

$A = \int_{-2}^{1} e^x+2\,dx = e^x + 2x \Big|_{-2}^{1} =$

$(e^1+2)-(e^{-2}-4) =$

$e^1 - e^{-2} + 6$

$e - \frac{1}{e^2} + 6$

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Panel 9

9. Suppose the marginal cost of making q throw rugs is $c' = 8q - 3\sqrt{q} + 4e^q$, and the fixed cost is \$4400. Find the formula for the cost function. (6%)

$$C(q) = \int 8q - 3\sqrt{q} + 4e^q \, dq =$$

$$8\frac{1}{2}q^2 - 3\frac{2}{3}q^{3/2} + 4e^q + C =$$

$$C(q) = 4q^2 - 2q^{3/2} + 4e^q + C.$$

$$C(q) = 4q^2 - 2q^{3/2} + 4e^q + 4396$$

Fixed cost: $C(0) = 4 + C \overset{\text{want}}{=} 4400$

$$C = 4400 - 4 = 4396$$

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Panel 10

Final Topics: Financial Mathematics

Recall Compound Interest Formula: If you invest a principal P at an interest rate r per period compounded for n periods in total, you have:

$$S = P(1+r)^n$$

Ex: \$1000 at rate 8% compounded quarterly for 5 years

$$\Rightarrow S = 1000\left(1 + \frac{0.08}{4}\right)^{5 \cdot 4} = 1000(1.02)^{20} = 1000 \cdot 1.486 = \$1486$$

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Panel 11

Ex: Suppose \$500 is compounded semi-annually over 3 years and amounts to \$588.38. What is the nominal interest rate?

$S = P(1+r)^n \Rightarrow$

$588.38 = 500\left(1+\frac{r}{2}\right)^6$

$1.1767 = \frac{588.38}{500} = \left(1+\frac{r}{2}\right)^6 \quad \Big|\sqrt[6]{}$

$\sqrt[6]{1.1767} = 1+\frac{r}{2}$

$1.0275 = 1+\frac{r}{2} \Rightarrow 0.0275 = \frac{r}{2} \Rightarrow 0.055 = r$

rate was 5.5%

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Panel 12

Ex: How long will it take for \$600 to amount to \$900 at APR of 6% compounded quarterly?

$S = P(1+r)^n$

$900 = 600\left(1+\frac{0.06}{4}\right)^n$

$\frac{900}{600} = (1.015)^n$

$1.5 = (1.015)^n \quad \Big| \ln$

$\ln(1.5) = \ln(1.015^n)$

$= n \ln(1.015) \Rightarrow n = \frac{\ln(1.5)}{\ln(1.015)} = \frac{0.40548}{0.0149} = 27.23$

It takes 28 periods, or 7 years (with 4 periods each)