Panel 1

Anti-Derivatives

$\int f(x)\,dx$ is that function $F(x)$ such that $F' = f$

'integral of f dx'

Ex: $\int x^2 + \frac{1}{x} + e^x \, dx = \frac{1}{3} x^3 + \ln(x) + e^x + C$

Rules:

$\int x^p \, dx = \frac{1}{p+1} x^{p+1} + C \qquad (p \neq -1)$

$\int \frac{1}{x} \, dx = \ln(x) + C \qquad \int e^x \, dx = e^x + C$

1

Panel 2

Ex: $\int 3x^3 + 4\sqrt{x} - \frac{7}{x^2} + \frac{3}{x} + 4e^x + 4 \, dx =$ ($\sqrt{x} = x^{1/2}$, $\frac{1}{x^2} = x^{-2}$, $\frac{1}{x} = x^{-1}$)

$= 3\frac{1}{4} x^4 + 4\frac{1}{3/2} x^{3/2} - 7\frac{1}{(-1)} x^{-1} + 3 \cdot \ln|x| + 4e^x + 4x + C$

$= \frac{3}{4} x^4 + \cancel{4} \cdot \frac{2}{3} x^{3/2} + 7 x^{-1} + 3\ln|x| + 4e^x + 4x + C$ (8; $7 x^{-1} = 7/x$)

Ex: $F(x) = \int x^2 + 2\,dx$ with $F(0) = 5 \quad \Rightarrow F(x) = \frac{1}{3} x^3 + 2x + 5$

$F(x) = \frac{1}{3} x^3 + 2x + C$ Want: $F(0) = 5 = C$

$P(x) = 5x + e^x - 11 \Rightarrow$

$P(3) = 15 + e^3 - 11$

Ex: Marginal Profit is $P'(x) = 5 + e^x$, and $P(0) = -10$. Find $P(3)$. $P(x) = \int 5 + e^x \, dx = 5x + e^x + C$, $P(0) = -10 = 1 + C$, $C = -11$

2

Panel 3

Name: ______

Quiz #8

Evaluate the following indefinite integrals:

a) $\int 2x \, dx$

b) $\int x^4 \, dx$

c) $\int 9 \sqrt[3]{x} + 3e^x \, dx$

d) $\int \frac{3}{2x^2} - \frac{7}{3} \sqrt{x} \, dx$

3

Panel 4

(2) Find a function $y(x)$ such that $y' = 4x - \frac{3}{x}$ and $y(1) = 2$

(3) If a marginal cost function is $C'(q) = 10q - 3q^2$ and the fixed cost is \$10, find the total cost for producing $q = 3$ items.

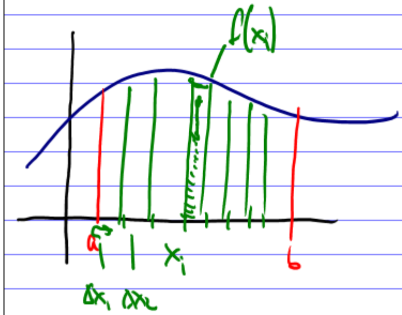
4

Panel 5

Definite Integral:

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} f(x_1)\Delta x_1 + \dots f(x_n)\Delta x_n$$

integral of f (from a to b)



(too much work for us)

Fundamental Theorem of Calculus: If f is continuous

then $$\int_a^b f(x)\,dx = F(x)\Big|_{x=a}^{x=b} = F(b)-F(a),$$

(F is antiderivative)

5

Panel 6

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$$

Ex: $$\int_0^2 2x\,dx = 2\tfrac{1}{2}x^2\Big|_0^2 = x^2\Big|_0^2 = (2)^2 - 0^2 = \underline{\underline{4}}$$

$$\int_0^1 5e^x - 7\sqrt[3]{x^5}\,dx = 5e^x - 7\tfrac{3}{8}x^{8/3}\Big|_0^1 =$$

$x^{5/3}$

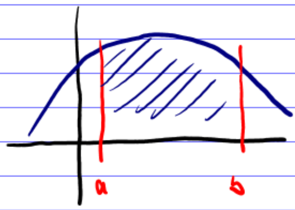
$$\left(5e^1 - 7\cdot\tfrac{3}{8}1\right) - \left(5e^0 - 7\tfrac{3}{8}\cdot 0\right)$$

$$5e - \tfrac{21}{8} - 5$$

6

Panel 7

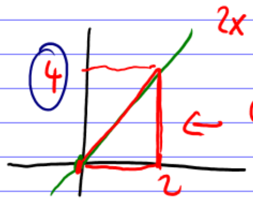
Interpretation of Definite Integral



$$\int_a^b f(x)\,dx = F(b) - F(a)$$

is area under $f(x)$, as long as $f(x) \geq 0$ (positive)

$$\int_0^2 2x\,dx = x^2\Big|_0^2 = (2)^2 - (0)^2 = \underline{\underline{4}}$$



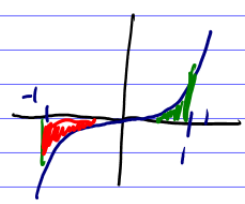
← area $= \frac{1}{2} b h = \frac{1}{2} \cdot 2 \cdot 4 = \underline{\underline{4}}$

$y = 2x$

7

Panel 8

Ex: $$\int_{-1}^{1} x^3\,dx = \frac{1}{4} x^4 \Big|_{-1}^{1} = \frac{1}{4}(1)^4 - \frac{1}{4}(-1)^4 = 0$$



x^3 is not always positive on $[-1,1] \Rightarrow$ "net" area of zero

Ex: $$\int_{-10}^{10} 5x - 7x^3 + 9x^5\,dx = 5\frac{1}{2}x^2 - 7\frac{1}{4}x^4 + 9\frac{1}{6}x^6 \Big|_{-10}^{10} =$$

$$= \frac{5}{2}x^2 - \frac{7}{4}x^4 + \frac{3}{2}x^6 \Big|_{-10}^{10} =$$

net area is zero

$$= \left[\frac{5}{2}(10)^2 - \frac{7}{4}(10)^4 + \frac{3}{2}(10)^6\right] - \left[\frac{5}{2}(-10)^2 - \frac{7}{4}(-10)^4 + \frac{3}{2}(-10)^6\right]$$

$= 0$

8

Panel 9

$\int f(x)\,dx$ = antiderivative $F(x)+C$ (indefinite integral)

$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b)-F(a)$ (definite integral)

Ex: $\int 3x^4 - \sqrt{x} + \frac{5}{x}\,dx = 3\frac{1}{5}x^5 - \frac{2}{3}x^{3/2} + 5\ln|x| + C$

$\int_1^2 x^3 - x^2\,dx = \frac{1}{4}x^4 - \frac{1}{3}x^3\Big|_1^2 = \left(\frac{1}{4}(2)^4 - \frac{1}{3}2^3\right) - \left(\frac{1}{4}1^4 - \frac{1}{3}1^3\right) =$

$= 4 - \frac{8}{3} - \frac{1}{4} + \frac{1}{3} =$

9