Panel 1

Had fun with Derivatives ....

1

Panel 2

Had fun with Derivatives ....

2

Panel 3

Anti-Derivatives

If f is a function, then any function F with $F' = f$ is called an antiderivative of f.

You write $F(x) = \int f(x)\,dx$

integral of ...

Ex: $\int 2x\,dx = x^2$ is an antiderivative

because $\frac{d}{dx}[x^2] = 2x$

Another antideriv. is x^2+7 or x^2-5 or x^2+C

constant

3

Panel 4

Ex:

$\int 1\,dx = \cancel{x^0 1}\ x^1 = x + C$, $\frac{d}{dx}[x+c] = 1$

$\int x\,dx = \cancel{x^1}\ \frac{1}{2}x^2 + C$, $\frac{d}{dx}[\frac{1}{2}x^2] = x$

$\int x^5\,dx = \frac{1}{6}x^6 + c$ ✓

$\int x^8\,dx = \frac{1}{9}x^9$

$\int \sqrt{x}\,dx = \int x^{1/2}\,dx = \frac{2}{3}x^{3/2} + c$

$\int \frac{1}{x^3}\,dx = \int x^{-3}\,dx = -\frac{1}{2}x^{-2} + C$

$\int \frac{1}{\sqrt[3]{x^4}}\,dx = \int x^{-4/3}\,dx = -3x^{-4/3+1} = -3x^{-1/3} + C$

4

Panel 5

More:

$$\int x^2 + 2x \, dx = \frac{1}{3}x^3 + 2\frac{1}{2}x^2$$

$$= \frac{1}{3}x^3 + x^2 + C$$

$$\int 2\sqrt[5]{x^4} - 7x^3 + 10 \, dx =$$

$$\int 2x^{4/5} - 7x^3 + 10 \, dx =$$

$$2\frac{5}{9}x^{9/5} - 7\frac{1}{4}x^4 + 10x + C$$

$$\frac{d}{dx}\left[\frac{10}{9}x^{9/5} - \frac{7}{4}x^4 + 10x + C\right] = \frac{10}{9}\cdot\frac{9}{5}x^{4/5} - \frac{7}{4}4x^3 + 10 -$$

$$2x^{4/5} - 7x^3 + 10$$

5

Panel 6

Even more:

① Find a function y such that $y' = 8x - 4$ and $y(2) = 5$

If $y' = 8x - 4$

then $y = \int 8x - 4 \, dx = 4x^2 - 4x - 3$

$$8\frac{1}{2}x^2 - 4x + C = 4x^2 - 4x + C$$

$$y(x) = 4x^2 - 4x + C$$

$$5 = y(2) = 4(2)^2 - 4(2) + C$$

$$= 16 - 8 + C$$

$$5 = 8 + C \Rightarrow C = -3$$

6

Panel 7

(i) Find a function y such that $y'' = x^2 - 6$ and also $y'(0) = 2$ and $y(1) = -1$

$$y'' = x^2 - 6 \Rightarrow y' = \int x^2 - 6 \, dx = \frac{1}{3}x^3 - 6x + C$$

$$\boxed{2 = y'(0) = C}$$

$$y' = \frac{1}{3}x^3 - 6x + 2$$

$$y = \int \frac{1}{3}x^3 - 6x + 2 \, dx = \frac{1}{3}\frac{1}{4}x^4 - 6\frac{1}{2}x^2 + 2x + D$$

$$= \frac{1}{12}x^4 - 3x^2 + 2x + D$$

$$-1 = y(1) = \frac{1}{12} - 3 + 2 + D, \quad D = -1 - \frac{1}{12} + 3 - 2 = -\frac{1}{12}$$

7

Panel 8

Ex: Suppose the marginal revenue function for a product is $\frac{dr}{dq} = 2000 - 20q - 3q^2$

Find the demand function $\quad R(q) = q \cdot P(q)$ demand

$$R'(q) = 2000 - 20q - 3q^2$$

$$R(q) = \int 2000 - 20q - 3q^2 \, dq =$$

$$= 2000q - 20\frac{1}{2}q^2 - 3\frac{1}{3}q^3 + C$$

$$= 2000q - 10q^2 - q^3 + C. \quad \text{Know } R(0) = 0 = C$$

$$R(q) = 2000q - 10q^2 - q^3 = q(2000 - 10q - q^2).$$

$$P(q) = 2000 - 10q - q^2$$

8

Panel 9

Rules of Integration

$$\int x^p\,dx = \frac{1}{p+1}x^{p+1} + C \qquad \frac{d}{dx}\left(x^p\right) = px^{p-1}$$

$$\int \frac{1}{x}\,dx = \ln(x) + C$$

$$\int e^x\,dx = e^x + C$$

sums / differences

$$\int f(x) \pm g(x)\,dx = \int f(x)\,dx \pm \int g(x)\,dx$$

9

Panel 10

e) $\int \frac{2x^2}{7} - \frac{8}{3}x^4\,dx$

$$\int \frac{2}{7}x^2 - \frac{8}{3}x^4\,dx = \frac{2}{7}\frac{1}{3}x^3 - \frac{8}{3}\frac{1}{5}x^5 + C = \frac{2}{21}x^3 - \frac{8}{15}x^5 + C$$

f) $\int (x^2+5)(x-3)\,dx$ FOIL, then integrate

g) $\int \frac{2}{\sqrt{x^3}} - \frac{\sqrt[4]{x^5}}{8}\,dx = \int 2x^{-3/2} - \frac{1}{8}x^{5/4}\,dx =$

$$= 2(-2)x^{-1/2} - \frac{1}{8}\frac{4}{9}x^{9/4} + C$$

h) $\int (3x+2)^3\,dx$ FOIL thrice, then integrate

10

Panel 11

a) $\int \frac{1}{x}\,dx = \int \cancel{x^{-1}}\,dx = \cancel{\frac{1}{0}x^{0}}$ nope $= \ln|x| + C$

b) $\int e^x\,dx = e^x + C$

c) $\int 5e^x + \frac{1}{3x}\,dx = 5e^x + \frac{1}{3}\cdot\ln|x| + C$

11

Panel 12

If the fixed costs for producing Acme Quality Widgets is \$2000 and the marginal cost is $c'(q) = 0.08q^2 - 1.6q + 6.5$, find the cost for producing 25 units.

$C'(q) = 0.08q^2 - 1.6q + 6.5$

$\Rightarrow C(q) = \int 0.08q^2 - 1.6q + 6.5\,dx =$

$= 0.08\frac{1}{3}q^3 - 1.6\frac{1}{2}q^2 + 6.5q + C$

Know: fixed cost is \$2000 $= C(0) = C$

$C(q) = \frac{0.08}{3}q^3 - 0.8q^2 + 6.5q + 2000$

$C(25) = \frac{0.08}{3}(25)^3 - 0.8(25)^2 + 6.5(25) + 2000$

Answer

12