

Panel 1

Different Terms for "Derivative":

Definition and various rules

Slope of tangent, rate of change, marginal cost

Higher Order derivatives f'' , f''' , $f^{(n)}$, $f^{(n)}$, $f^{(n)}$

$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} \ln(x) = \frac{1}{x}$$

inc/decreasing (+ concave up/down)

max/min

1

Panel 2

Name: _____

Quiz

① Find f' if $f(x) = 3x^2 - 7e^x + 5 \ln(x) + \sqrt{x} + \pi$

② Find f''' if $f(x) = 3x^2 - 7x + 9$

2

Panel 3

Take $R(q) = 2q^3 - 3q^2 + 3$ is revenue

Say I produce 5 items how much revenue would the 6th item bring?

A) $R(6) - R(5)$ or indeed marginal revenue:

$$R'(q), R'(q) = 6q^2 - 6q \text{ at } q=5, R'(5) = 150 - 30 = \underline{\underline{120}}$$

$$R(6) = 2 \cdot 32 - 108 + 3 = \underline{\underline{323}}$$

$$R(5) = 250 - 75 + 3 = \underline{\underline{178}}$$

Find max/min Revenue.

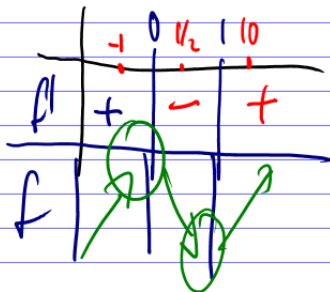
3

Panel 4

$$R(q) = 2q^3 - 3q^2 + 3$$

$$R'(q) = 6q^2 - 6q = 0$$

$$6q(q-1) = 0 \Rightarrow q=0, q=1$$



$q=0$ yields max revenue

$q=1$ yields min. revenue

4

Panel 5

$$f(x) = 7e^x + \frac{3}{\sqrt[3]{x^2}} + \pi - 7 \ln(x) + \frac{10}{x^4} \quad (+1)$$

Find f'' :

$$f'(x) = 7e^x + \frac{2}{3}x^{-1/3} - \frac{7}{x} - 40x^{-5}$$

$$f''(x) = 7e^x + \frac{2}{3} \left(\frac{1}{3}\right) x^{-4/3} + 7x^{-2} + 200x^{-6}$$

5

Panel 6

- ③ If $P(x) = -2x^3 + 3x^2 - 6$ is profit function, is profit increasing or decreasing at $x=2$?
- ④ Find all critical points for $f(x) = x^4 - 12x^2$
- 6

Panel 7

First vs Second Derivative

f' deriv $\begin{cases} \text{inc/decr} \\ \text{max/min} \\ \text{rate of change} \end{cases}$

also, f is distance
 $\Rightarrow f' = \text{speed}$
 $f'' = \text{acceleration}$

$f'' = \text{second deriv is rate of change of } f'$

$f'' > 0 \Rightarrow f$ is concave down \cap
 $f'' < 0 \Rightarrow f$ is concave up \cup

$\left. \begin{array}{l} \cap \\ \cup \end{array} \right\} \text{ curvature}$

7

Panel 8

Ex: Suppose $s(t) = -16t^2 + 10t + 5$ is the dis
 function for some object. Find his

a) velocity and acceleration after 10 seconds

$v(t) = s'(t) = -32t + 10 \Rightarrow v(10) = -32 \cdot 10 + 10 = \underline{\underline{-310}}$

$a(t) = s''(t) = -32 \Rightarrow a(10) = \underline{\underline{-32}}$

b) When is the velocity zero? Interpret!

$v(t) = -32t + 10 = 0$

$t = \frac{10}{32} \Rightarrow$ is zero velocity \Rightarrow at highest point

8

Panel 9

Meaning of f''

$f'' > 0 \Rightarrow \cup$ (think x^2)

$f'' < 0 \Rightarrow \cap$ (think $-x^2$)

think of $f(x) = x^2$

Ex: Investigate concavity of $f(x) = 2x^3 - 6x^2$

$f'(x) = 6x^2 - 12x$

$f''(x) = 12x - 12 = 0$

$12(x-1) = 0, x=1$

possible point of inflection

	0	2
f''	-	+
f	\cap	\cup

Panel 10

Questions about increasing/decreasing/max/min

1. Find f'
2. Solve $f' = 0$ (critical points)
3. Set up table f' and f

Questions about concavity/inflection points

- 1.) Find f''
- 2.) Set $f'' = 0$ (possible inf. points)
- 3.) Set up a table with f'' and f

Panel 11

$$f(x) = x^3 - 9x^2 + 15x - 4$$

① Decide where f is incr./decr./max/min

$$f'(x) = 3x^2 - 18x + 15 = 0$$

$$3(x^2 - 6x + 5) = 0$$

$$\underline{3(x-5)(x-1) = 0}$$

$x=1, x=5$ are the critical points : f has max at $x=1$

and a min at $x=5$

	0	1	5	
f'		+	-	+
f		↗	↘	↗

11

Panel 12

$$f(x) = x^3 - 9x^2 + 15x - 4$$

② Decide where f is concave up/down

$$f''(x) = 3x^2 - 18x + 15$$

$$f''(x) = 6x - 18 = 0, x=3 \text{ possible inf. point.}$$

	3	4
f''	-	+
	∩	∪

12

Panel 13

Ex: $f(x) = x^2 - 50 \cdot \ln(x), x > 1$

13

Panel 14

Sketch graph of $f(x) = 2x^3 + 3x^2 - 12x - 3$

① Find f' and critical
pts:

$$f'(x) = 6x^2 + 6x - 12 = 6(x^2 + x - 2) = 6(x+2)(x-1) = 0$$

$x = -2, x = 1$ critical

② Find f'' and poss.
inflection pts

$$f''(x) = 12x + 6 = 0 \quad x = -\frac{1}{2} \text{ poss. inf. pt}$$

③ Make a big table.

	-3	-2	-1	$-\frac{1}{2}$	0	1	10
f'	+	-	-	-	-	+	+
f''	-	-	-	+	+	+	+

↓

next time

14

Panel 15

Recipe for Curve Sketching : $f(x) = x^3 - 9x^2 + 15x - 4$

① Find f'
Solve $f' = 0$

② Find f''
Solve $f'' = 0$

③

f'		
f''		
f		

④ Find $f(x)$ at
special points.

15