Panel 1

The (Calc) Story so far....

$\lim_{x \to a} f(x) = L$ incl. $\lim_{x \to \pm\infty}$ and $\lim f$, $f = \begin{cases} - \\ - \end{cases}$

f continuous at $x = a$ (a) $f(a)$

(b) $\lim_{x \to a} f(x) =$ exists

(c) agree

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

slope of tangent, inst. rate of change, marginal cost revenue profit

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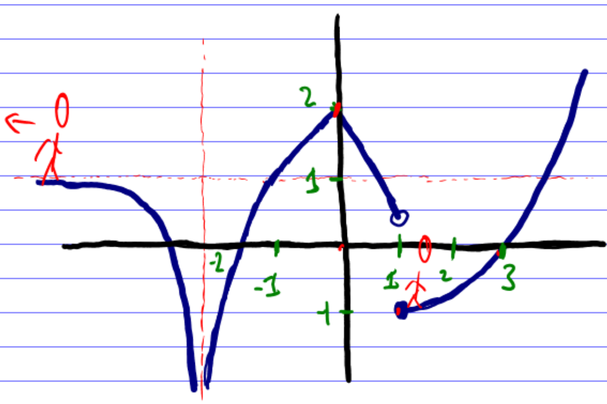
Panel 2



$f(3) = 0$

$f'(3) > 0$

$\lim_{x \to 1^+} f(x) = -1$

$\lim_{x \to -\infty} f(x) = 1$

List points where f is <u>not</u> continuous : -2 and 1

List points where f is <u>not</u> differentiable -2, 1, 0

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Panel 3

Differentiation Shortcuts

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

f' is the slope of the tangent line

Power Rule: $\frac{d}{dx}[x^{n}] = n x^{n-1}$

Sum Rule: $\frac{d}{dx}(f(x) \pm g(x)) = f'(x) \pm g'(x)$

Constant Factor: $\frac{d}{dx}(c \cdot f(x)) = c f'(x)$

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Panel 4

More Examples:

A) $f(x) = x^5$ $\qquad f'(x) = 5x^4$

B) $f(x) = x^3 + 9x^0$ $\qquad f'(x) = 3x^2 + 0 = 3x^2$

C) $f(x) = 3x^2 - 7x$ $\qquad f'(x) = 6x - 7$

D) $f(x) = 1/x^3 = x^{-3}$ $\qquad f'(x) = -3x^{-4}$

E) $f(x) = 3x^2 - \frac{7}{x} + \sqrt{x} = 3x^2 - 7x^{-1} + x^{1/2}$

$f'(x) = \quad = 6x + 7x^{-2} + \frac{1}{2}x^{\cancel{1/2-1}\, -1/2}$

F) $f(x) = 9x^3 - 8\sqrt{x^3} + 5\sqrt[3]{x^2} + \pi$

$9x^3 - 8x^{3/2} + 5x^{2/3} + \pi$

$f'(x) = 27x^2 - 8 \cdot \frac{3}{2} \cdot x^{3/2-1} + 5\frac{2}{3}x^{2/3-1} = 27x^2 - 12x^{1/2} + \frac{10}{3}x^{-1/3}$

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Panel 5

Ex: $f(x) = x^4(3x^2 - 2x + 1) =$
$= 3x^6 - 2x^5 + x^4 \Rightarrow f'(x) = 18x^5 - 10x^4 + 4x^3$

$g(x) = (2x-3)^2 = 4x^2 - 12x + 9$
$g'(x) = 8x - 12$

$(a+b)^2 = a^2 + 2ab + b^2$
$(a-b)^2 = a^2 - 2ab + b^2$
$(a+b)(a-b) = a^2 - b^2$

$h(x) = \left(x - \frac{1}{x}\right)^2 = x^2 - 2 + \frac{1}{x^2}$

$h'(x) = 2x - 2x^{-3}$

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Panel 6

More complicated differentiation Rules

Product Rule: $\frac{d}{dx}(f(x) \cdot g(x)) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

Quotient Rule: $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x) g(x) - f(x) \cdot g'(x)}{[g(x)]^2}$

Bad news: complicated

Chain Rule: $\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$

good news: Skip it! (or use computer)

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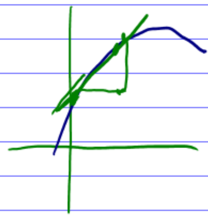
Panel 7

Why study Derivatives



$f'(x) = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \text{rate of change}$

$\frac{f(x+h)-f(x)}{h}$ = avg. rate of change

$f'(x)$ = instantaneous rate of change

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Panel 8

Ex: Suppose the position function of an object is given by $f(t) = 3t^2 + 5$

a) Find avg. rate of change over [2,3]

$$\frac{f(3)-f(2)}{3-2} = \frac{32-17}{1} = \frac{15}{1} = 15$$

b) how fast is the car going when $t=2$?

$$f'(2) = 6t\Big|_{t=2} = 12$$

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Panel 9

Ex: Let $p = 100 - q^2$ be a demand function.
How fast is price changing when $q = 5$?

$$p'(5) = p'(q)\Big|_{q=5} = -2q\Big|_{q=5} = -10$$

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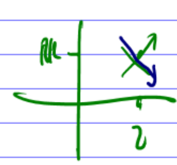
Panel 10

Ex: Suppose revenue function is $R(q) = 9q - q^3$ and the current level of production is at $q = 2$ (thousand). Should you change the production level up (produce more items) or down (produce fewer items)?

$$R'(q) = 9 - 3q^2 \Rightarrow R'(2) = 9 - 12 = \underline{-3}$$

rate of change is neg. at $q = 2 \Rightarrow R$ is decreasing at $q = 2$

Decrease production will get higher revenue



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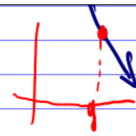
Panel 11

Say $R(q)$ is s.t. $R'(10) < 0 \Rightarrow$  Decrease production gets higher revenue

$R'(10) > 0 \Rightarrow$  Increase production

$R'(10) = 0 \Rightarrow$ Do nothing

Say $C(q)$ is s.t. $C'(10) < 0 \Rightarrow$  Incr. production will lower cost

$C'(10) > 0 \Rightarrow$  Produce less

$C'(10) = 0 \Rightarrow$ Do nothing

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Panel 12

Marginal Cost: The marginal cost is the approx. cost of producing one additional unit.

Marginal Cost is $C'(q)$ (derivative of C)

Ex: A cost function is $C(q) = 0.0001 q^3 - 0.02 q + 5q + 5000$

Find fixed cost and marginal cost when $q = 50$

Fixed cost: $C(0) = \$5000$

Marginal cost: $C'(q)\Big|_{q=50} = 0.0003 q^2 - 0.02 \cdot 1 + 5 = 0.0003 q^2 + 4.98 \Big|_{q=50} = 5.73$

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Panel 13

Relative Rate of Change

If $f(x)$ is a function, rate of change is

The rate of change relative to $f(x)$ is called the relative rate of change

Ex: Find relative rate of change for $y = 3x^2 - 5x + 25$ when $x = 5$.

nothing

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Panel 14

Final example: Suppose a revenue function is

$R(q) = \frac{2q^2+20}{q+1}$. For what level of production is the marginal revenue zero?

$R'(q) = 0$ solve for q.

$$R'(q) = \frac{2(q^2+2q-10)}{(q+1)^2} = 0$$

$q^2+2q-10=0$, $q = 2.31662$ and $-\cancel{4.31}$

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Panel 15

Derivatives of special functions

If $f(x) = e^x$ then

next time

quiz on Wed.

Exam 2 coming up soon.

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