Panel 1

Last Time

limits at infinity: What happens to $f(x)$ as x gets larger + larger

limits at infinity of rational functions:

$$\lim_{x\to\pm\infty} \frac{p(x)}{q(x)} = \begin{cases} \deg(p) = \deg(q): & \#/\# \text{ (coef. for highest degree)} \\ \deg(p) < \deg(q): & 0 \\ \deg(p) > \deg(q): & \pm\infty \end{cases}$$

Continuity:

(1) $f(c)$ exists (2) $\lim_{x\to c} f(x)$ exists (3) (1) = (2)

Graphically: no holes, no gaps

1

Panel 2

Examples:

a) $\lim_{x\to\infty} \frac{7x - 5 - 3x^2}{2x^2 + 9x + 7} = -3/2$

b) $\lim_{x\to -\infty} \frac{x^2 - 7x - 9}{3x^3 - 2x^2 - x} = 0$

c) $\lim_{x\to -\infty} \frac{2x^5 - 3x}{3x^3 - 2x^4} = \pm\infty$

$\sim \lim_{x\to -\infty} -x = +\infty$

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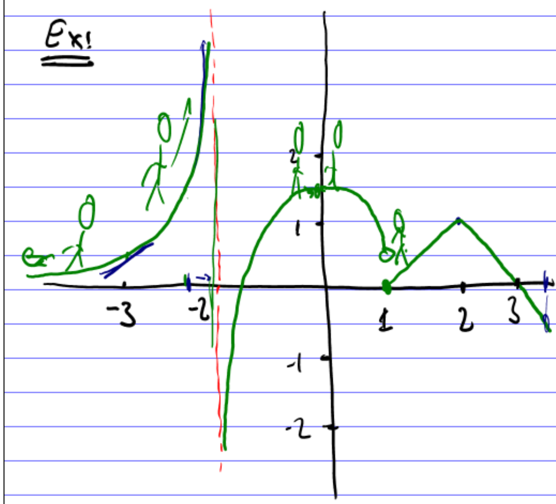
Panel 3

Ex!



a) $\lim_{x \to 3} f(x) = 0$

b) $\lim_{x \to 1^+} f(x) = 0$

c) $\lim_{x \to -2^-} f(x) = \infty$

d) $\lim_{x \to -\infty} f(x) = 0$

e) $\lim_{x \to 0} f(x) = 1.5$

g) $\lim_{x \to 1} f(x) =$ dne.

Where is $f(x)$ not continuous? at $x=-2$ and 1

3

Panel 4

Continuity is a property that most functions (defined in one line) automatically have.

Every polynomial is cont.

Every rational function is cont. wherever it is defined

$\lim_{x \to 1} 5x - 1 = 4$

$f(x) = \frac{x-7}{x^2-1}$

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Panel 5

Ex:

$$f(x) = \begin{cases} \frac{x^2-9}{x-3} & \text{if } x \neq 3 \\ \cancel{k}\,9 & \text{if } x = 3 \end{cases}$$

Find value of k so that f is continuous at $x=3$

(1) $f(3) = \cancel{k}\,9$

(2) $\lim_{x \to 3} \frac{x^2-9}{x-3} = \lim_{x\to 3} \frac{(x+3)\cancel{(x-3)}}{\cancel{x-3}} = \lim_{x \to 3} x+3 = 6$

(3) (1) = (2)

$k = 6$ Take $k=6$ will make f cont. at $x=3$

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Panel 6

Continuity is a property that most functions (defined in one line) automatically have.

Every polynomial

Every rational function

$\lim_{x \to 1} 5x - 1 =$

6

Panel 7

Applications of Limits

Ex: The population of a small city is $P(t) = 50{,}000 - \frac{2000}{t+1}$.

a) What is the current population

b) What is the population in the long run?

a) $P(0) = 49000$ b) $\lim_{t\to\infty} 50000 - \frac{2000}{t+1} = 50000 - 0$

Ex: For a host-parasite relation it was determined that when the host density is x (# of hosts per area) the number of host parasites is $y = \frac{900x}{10+45x}$ $\lim_{x\to\infty} \frac{900x}{10+45x} = \frac{900}{45} = \underline{\underline{20}}$

If hosts increase without bound, what about parasites?

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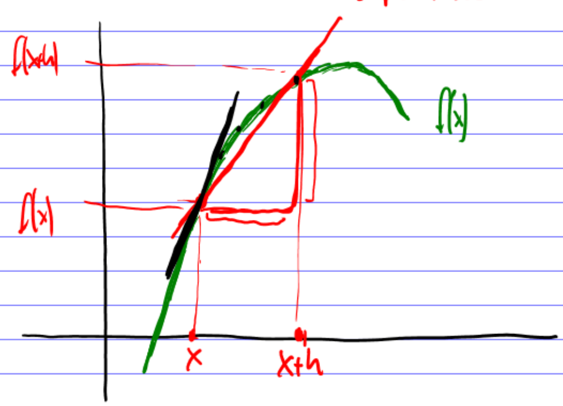
Panel 8

The Most Famous Limit of All:

slope of secant line is $\frac{f(x+h)-f(x)}{h}$



$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \frac{0}{0}$$

slope of tangent line

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Panel 9

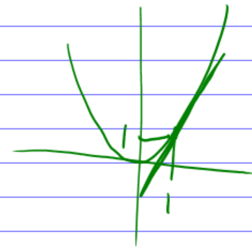
The Derivative

If $f(x)$ is a function, we define the derivative of f as

$$\overset{\text{prime}}{f'(x)} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

provided the limit exists.

Geometrically,



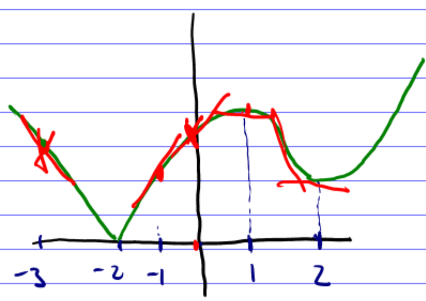
Ex: $f(x) = x^2$. Find $f'(x) = 2x$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h\to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h} =$$

$$= \lim_{h\to 0} \frac{h(2x+h)}{h} = \lim_{h\to 0} 2x+h = 2x$$

Panel 10

Ex: Find the indicated quantities: (signs only)



$f'(-3) < 0$

$f'(-1) > 0$

$f'(0) > 0$

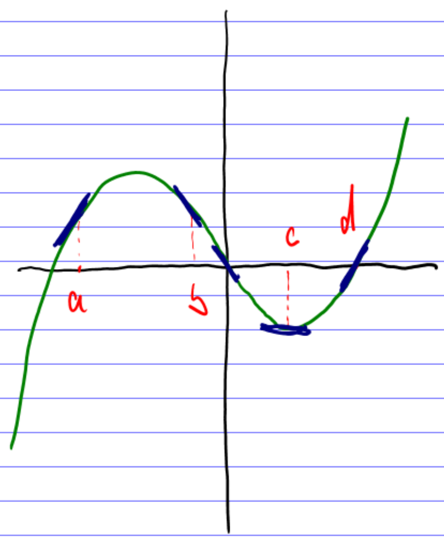
$f'(1) = 0$

$f'(2) = 0$

$f'(1.5) < 0$

Panel 11

Ex:



Estimate the signs of

1) $f'(a) > 0$

2) $f'(b) < 0$

3) $f'(0) < 0$

4) $f'(c) = 0$

5) $f'(d) > 0$

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Panel 12

Ex: Find the derivative to $f(x) = 2x^2 - x + 1$ at the point $x = 1$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{[2(x+h)^2 - (x+h) + 1] - [2x^2 - x + 1]}{h} =$$

$$= \lim_{h\to 0} \frac{2(x^2+2xh+h^2) - x - h + 1 - 2x^2 + x - 1}{h} =$$

$$= \lim_{h\to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 - \cancel{x} - h \cancel{+1} - \cancel{2x^2} \cancel{+x} \cancel{-1}}{h} = \lim_{h\to 0} \frac{4xh + 2h^2 - h}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(4x + 2h - 1)}{\cancel{h}} =$$

$$= \lim_{h\to 0} 4x + 2h - 1 = \underline{\underline{4x - 1}}$$

$f'(x) = 4x - 1$

$\Rightarrow f'(1) = 4 - 1 = \underline{\underline{3}}$

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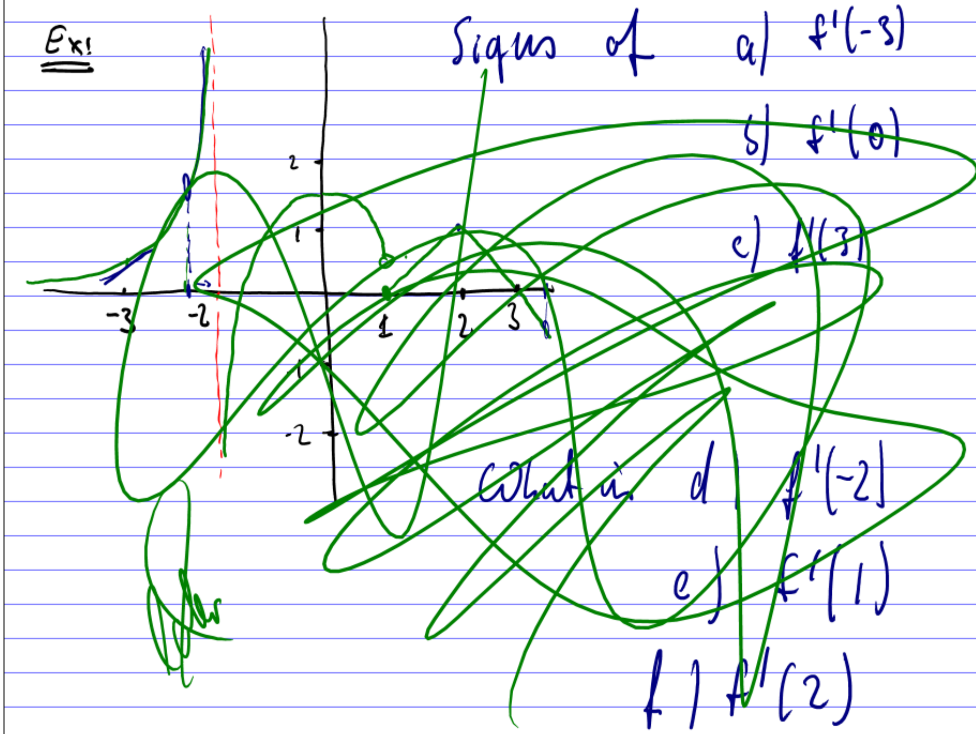
Panel 13

Ex!

Signs of a) $f'(-3)$

b) $f'(0)$

c) $f'(3)$



What is d) $f'(-2)$

e) $f'(1)$

f) $f'(2)$

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Panel 14

Find $\frac{d}{dx} f$, where $f(x) = \frac{1}{x}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h\to 0} \frac{\frac{x-(x+h)}{(x+h)x}}{h} =$$

$$= \lim_{h\to 0} \frac{-\cancel{h}}{(x+h)\cdot x} \cdot \frac{1}{\cancel{h}} = \lim_{h\to 0} \frac{-1}{(x+h)x} = -1/x^2$$

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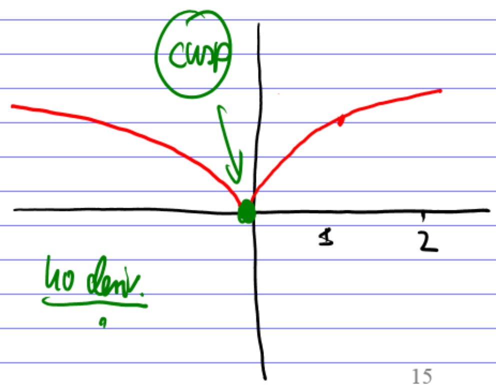
Panel 15

Derivatives are defined via limits. Thus, they do not necessarily have to exist:

Def: If the graph of a function does not have a unique tangent line at a point, it is not differentiable at that point.

Ex:



a) $f'(2) > 0$

b) $f'(1) > 0$

c) $f'(0)$ d.n.e

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Panel 16

Derivatives as limits can get complicated ⇒ need short cut:

Ex: $f(x) = x^1 \Rightarrow f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{x+h-x}{h} = 1 x^0$

$f(x) = x^2 \Rightarrow f'(x) = 2x^1$

$f(x) = x^3 \Rightarrow f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = 3x^2$

$f(x) = x^4 \Rightarrow f'(x) = 4x^3$

$f(x) = x^n \Rightarrow f'(x) = n x^{n-1}$

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Panel 17

Rules for Differentiation - Part I

The Power Rule: If $f(x) = x^n$ then $f'(x) = n x^{n-1}$

(use '$\frac{d}{dx}$' for derivative: $\boxed{\frac{d}{dx} x^n = n x^{n-1}}$

The Constant Rule:

$$\frac{d}{dx}[c] = 0$$

The Constant Factor Rule:

$$\frac{d}{dx} c \cdot f(x) = c\, f'(x)$$

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Panel 18

Examples: $f(x) = 3 \Rightarrow f'(x) = 0$

$f(x) = \frac{1}{x} = x^{-1}$ $\qquad g(x) = x^2 \Rightarrow f'(x) = 2x^1$

$f'(x) = -1 x^{-2}$ $\qquad h(x) = 3x^5 \Rightarrow f'(x) = 3 \cdot 5 x^4$

$= -\frac{1}{x^2}$ $\qquad k(x) = \sqrt{x} = x^{1/2} \Rightarrow f'(x) = \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}}$

$l(x) = \frac{9}{5} \sqrt[3]{x} = \frac{9}{5} x^{1/3} \Rightarrow f'(x) = \frac{9}{5} \cdot \frac{1}{3} x^{1/3-1} = \frac{3}{5} x^{-2/3}$

$m(x) = \frac{4}{\sqrt[3]{x^2}} = 4x^{-2/3} \Rightarrow f'(x) = 4 \cdot \left(-\frac{2}{3}\right) x^{-2/3-1} = -\frac{8}{3} x^{-5/3}$

$n(x) = 3\pi e^2$? $\Rightarrow n'(x) = 0$

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Panel 19

More (simple) Differentiation Rules

The Sum / Difference Rule: "Life is good" or "it works as it should"

$$\frac{d}{dx}\left[f(x) \pm g(x)\right] = f'(x) \pm g'(x)$$

Ex: $f(x) = 5x^2 + 3x - 7$

$f'(x) = 2x + 3 \cdot 1 x^0 = 2x + 3$

$g(x) = 5x^2 - \frac{7}{x^2} + 9 \sqrt[3]{x^4} + \pi^2$

$-7x^{-2}$

$9x^{4/3}$

$g'(x) = 10x + 14x^{-3} + 9 \cdot \frac{4}{3} x^{4/3 - 1} + 0$

$= 10x + 14x^{-3} + 12x^{1/3}$

No class Wed Oct 19

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