Panel 1

Last Time:

$\lim_{x \to a} f(x) = L$ means: as x gets close to a, $f(x) \to L$

$\lim_{x \to a^+} f(x)$ x is close to a, but $x > a$

$\lim_{x \to a^-} f(x)$ x close to a, but $x < a$

Graphical Limits

1

Panel 2

Ex: Find the following limits:

$\lim_{x \to 0} \frac{x^2 - 2x + 1}{x^2 - 4} = -\frac{1}{4}$

$\lim_{x \to 2} \frac{x^2 + 2x - 8}{x^2 - 4} = \lim_{x \to 2} \frac{(x+4)\cancel{(x-2)}}{(x+2)\cancel{(x-2)}} = \frac{6}{4} = \frac{3}{2}$

$\lim_{x \to 1^+} f(x) = 3$, where $f(x) = \begin{cases} x^2 - 1 & \text{if } x < 1 \\ 3x & \text{if } x > 1 \end{cases}$

$\lim_{x \to 1^+} (3x) = 3$

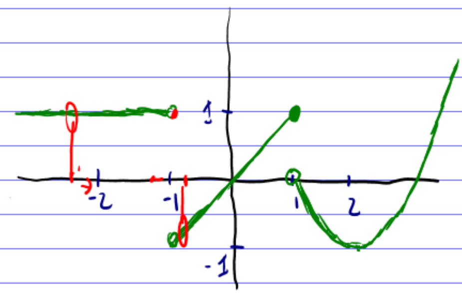
$\lim_{x \to 1} f(x) =$ undefined.

$\lim_{x \to 1^-} x^2 - 1 = 0$

2

Panel 3

Ex: Consider the graph of the function shown below



a) $\lim_{x \to -2} f(x) = 1$

b) $\lim_{x \to 0^+} f(x) = 0$

c) $\lim_{x \to -1^-} f(x) = 1$

d) $\lim_{x \to 1^+} f(x) = 0$

e) $\lim_{x \to 0} f(x) = 0$

f) $\lim_{x \to -1} f(x) = \underline{d.n.e.}$

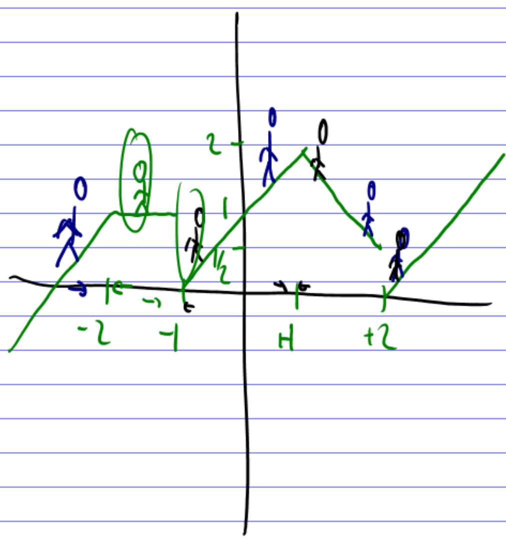
g) $\lim_{x \to 1^+} f(x) = 0$

$\lim_{x \to 1} f(x) = d.n.e.$

$\lim_{x \to 1^-} f(x) = 1$

3

Panel 4



$\lim_{x \to -2^+} f(x) = 1$

$\lim_{x \to -1^-} f(x) = 1$

$\lim_{x \to 1^+} f(x) = 2$

$\lim_{x \to 2^-} f(x) = 1/2$

4

Panel 5

Name: ____________

Quiz #4

① Find the following limits

a) $\lim_{x \to 0} \frac{x^2 - 5x + 4}{x - 2}$

b) $\lim_{x \to 2} \frac{x^2 - 3x + 2}{x^2 - 4}$

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Panel 6

② Suppose $f(x) = \begin{cases} x^2 + 3 & \text{if } x > 0 \\ 3x - 1 & \text{if } x < 0 \end{cases}$ Then

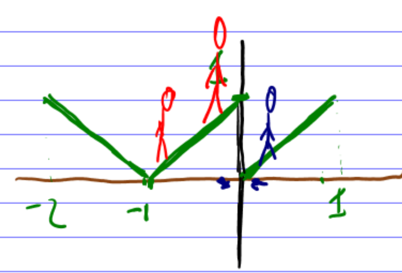
find $\lim_{x \to 0^+} f(x)$ if possible

$\lim_{x \to 0} x^2 + 3 = 3$

③ For f as shown, find

a) $\lim_{x \to 0^-} f(x)$

b) $\lim_{x \to -1^+} f(x)$



6

Panel 7

Limits at Infinity

Def: $\lim_{x\to\infty} f(x) = L$ means: as x gets bigger and bigger, and bigger, $f(x)$ gets close to L

Ex: $\lim_{x\to\infty} \frac{1}{x^2} = 0$

$\lim_{x\to-\infty} x^3 = -\infty$ or d.n.e. as a number

$\lim_{x\to-\infty} \frac{1}{x} = -0 = 0$

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Panel 8

Limits at Infinity for Rational Functions

(factor highest power)

$$\lim_{x\to\infty} \frac{x^3-2x-7}{3x^3+2} = \lim_{x\to\infty} \frac{\cancel{x^3}\left(1-\frac{2}{x^2}-\frac{7}{x^3}\right)}{\cancel{x^3}\left(3+\frac{2}{x^3}\right)} = \frac{1}{3}$$

$$\lim_{x\to\infty} \frac{x^2-7-x}{x^3-7x+9} = \lim_{x\to\infty} \frac{x^2\left(1-\frac{7}{x^2}-\frac{1}{x}\right)}{x^3\left(1-\frac{7}{x^2}+\frac{9}{x^3}\right)} = \frac{1}{x}\cdot 1 \to 0$$

$$\lim_{x\to\infty} \frac{x^4-7}{x^3-8x^2+9x} = \infty$$

Rule: Think of it as a race - highest power wins

8

Panel 9

Limits at Infinity:

$$\lim_{x\to\pm\infty} \frac{p(x)}{q(x)} = \begin{cases} \frac{\#}{\#} & \text{if } \deg(p)=\deg(q) \quad \text{(highest coeff.)} \\ 0 & \text{if } \deg(p)<\deg(q) \\ \pm\infty & \text{if } \deg(p)>\deg(q) \end{cases}$$

Ex:

$$\lim_{x\to-\infty} \frac{x^2-7x+9}{3-4x^2} = \frac{\cancel{x^2}\left(1-\cancel{\frac{7}{x}}+\cancel{\frac{9}{x^2}}\right)}{\cancel{x^2}\left(\cancel{\frac{3}{x^2}}-4\right)} = -\frac{1}{4}$$

$$\lim_{x\to\infty} \frac{3x^2-7}{9x^3-5} = 0$$

$$\lim_{x\to-\infty} \frac{3-4x+5x^3}{7x^2-9} = \pm\infty$$

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Panel 10

$$\lim_{x\to\infty} \frac{3x+7x^3+9}{4x-5+2x^3} = \frac{7}{2}$$

$$\lim_{x\to\infty} \frac{7-x-2x^2}{x^3+2x+9} = 0$$

$$\lim_{x\to\infty} \frac{x^2-1}{x^2+1} = 1$$

$$\lim_{x\to-\infty} \frac{x^4-7x+9}{x^3+2x+1} = \pm\infty$$

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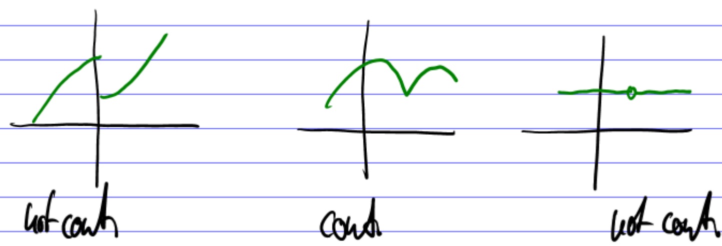
Panel 11

Continuity

Most simple functions have a graph that you can draw without lifting the pen $\Rightarrow$ Continuous



Def: f is continuous at $x=c$ if:

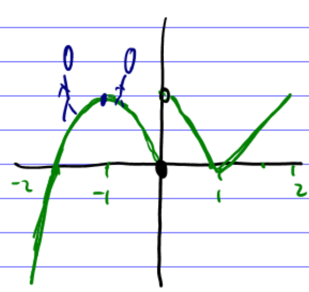
(1) $f(c)$ exist

(2) $\lim\limits_{x\to c} f(x)$ exists

(3) (1)=(2)

Panel 12

Ex:



$\lim\limits_{x\to -1} f(x) = 1$

$\lim\limits_{x\to 0} f(x) = \text{d.n.e.}$

$\lim\limits_{x\to 1} f(x) = 0$

Is f continuous at:

$x=-1$ YES

$x=0$ No

$x=1$ YES

Panel 13

Ex: $f(x) = \begin{cases} 1 & \text{if } x < 0 \\ -1 & \text{if } x = 0 \\ x^2 & \text{if } x > 0 \end{cases}$ Is f cont. at $\underline{x=0}$?

① $f(0) = -1$

② $\lim_{x \to 0} f(x) = \underline{\text{dne}} \begin{cases} \lim_{x \to 0^+} f(x) = 0 \\ \lim_{x \to 0^-} f(x) = 1 \end{cases}$ $\underline{\text{Not}}$ cont. at $x=0$

$f(x) = \begin{cases} x+6 & \text{if } x \geq 3 \\ x^2 & \text{if } x < 3 \end{cases}$ Cont. at $x=3$

HW