Panel 1

New Subject Entirely: Calculus (chapter 10 – 14)

Want to look at $f(x) = \frac{x^2-1}{x-1}$

Wonder: what happens if x is close to 1 (but not equal)

Note: Domain is all # but x=1

x	f(x)	x	f(x)
0.9	1.9	1.1	2.1
0.99	1.99	1.01	2.01
0.999	1.999	1.001	2.001
⋮	⋮	⋮	⋮

Pattern: as x gets closer to 1, either from above or below, f(x) gets closer to 2.

Call this ...

1

Panel 2

Def of Limit: $\lim_{x\to a} f(x) = L$ (limit as x approaches a equals L)

means as x gets closer and closer to a, f(x) gets closer to L.

Ex: $\lim_{x\to 1} \frac{x^2-1}{x-1} = 2$

$\lim_{x\to 1} 2x+3 = 5$ (cheated: plug in 1)

$\lim_{x\to 0} \frac{x^2-4}{x-2} = \frac{-4}{-2}$ (cheated: plug in 0)

$\lim_{x\to 2} \frac{x^2-4}{x-2} = \lim_{x\to 2} \frac{(x+2)\cancel{(x-2)}}{\cancel{x-2}} = \underline{4}$

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Panel 3

Ex: $\lim_{x\to 0} \frac{x^2-3x-4}{x^2-4x-5} = \frac{-4}{-5} = \underline{\underline{4/5}}$

$\lim_{x\to -1} \frac{x^2-3x-4}{x^2-4x-5} = \lim_{x\to -1} \frac{\cancel{(x+1)}(x-4)}{\cancel{(x+1)}(x-5)} = \frac{-5}{-6} = \underline{\underline{5/6}}$

would also come out using a table

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Panel 4

Limits may or may not exist!

$\lim_{x\to 0} \frac{1}{x} =$ does not exist

x	$f(x)$	x	$f(x)$
0.1	10	-0.1	-10
0.01	100	-0.01	-100
0.001	1000	-0.001	-1000
0.0001	10000	⋮	⋮
⋮			

Note

$\frac{0}{\#} = 0$ $\quad \lim_{x\to 0} \frac{x}{x-1} = 0$

$\frac{\#}{0} =$ undef. $\quad \lim_{x\to 0} \frac{x-1}{x} =$ undef

$\frac{0}{0} =$ more work $\quad \lim_{x\to 1} \frac{x^2-1}{x-1} = ?$

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Panel 5

How to find limits:

$\lim_{x \to a} f(x)$: what happens to $f(x)$ as x gets closer and closer to a, but is not equal to a.

(1) Try to substitute a for x, i.e. plug in a.

(2) # - answer

$\frac{0}{\#} = 0$ answer

$\frac{\#}{0}$ = undefined answer

$\frac{0}{0}$ = too bad, more work

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Panel 6

Find the following limits:

a) $\lim_{x \to 0} \frac{x-1}{x}$ = undefined $\left(\frac{\#}{0}\right)$

b) $\lim_{x \to 1} \frac{x-1}{x} = 0 \quad \left(\frac{0}{1}\right)$

c) $\lim_{x \to 0} \frac{x-1}{x^2-1} = \frac{-1}{-1} = 1$

d) $\lim_{x \to 1} \frac{x-1}{x^2-1} = \frac{0}{0} = \lim_{x \to 1} \frac{1\cancel{(x-1)}}{(x+1)\cancel{(x-1)}} = \frac{1}{2}$

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Panel 7

Today: $\lim_{x\to 0} \frac{e^{2x}-1}{x} = \frac{0}{0} = 2$

try 0.0001 ; ~~-0.999~~ 2.0002

try 0.000001 : 2.000002

$\lim_{x\to 2} \frac{x^2-3x+2}{x^2-4} = \frac{0}{0} = \lim_{x\to 2} \frac{\cancel{(x-2)}(x-1)}{\cancel{(x-2)}(x+2)} = \frac{1}{4}$

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Panel 8

One-Sided Limit

Say we have $f(x) = \begin{cases} 3x^2 & \text{if } x<0 \\ 2x & \text{if } x\geq 0 \end{cases}$ Then, clearly

$\lim_{x\to 2} f(x) = 4$ and $\lim_{x\to -2} f(x) = 12$

$0 = \lim_{x\to 0} f(x)$: $\begin{cases} \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} 2x = 0 \\ \lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} 3x^2 = 0 \end{cases}$

Def: $\lim_{x\to a^+} f(x)$ means x is close to a, but bigger than a

$\lim_{x\to a^-} f(x)$ means x is close to a, but less than a

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Panel 9

Theorem: $\lim_{x \to a} f(x) = L$ if and only if

$\lim_{x \to a^-} f(x) = L$ and $\lim_{x \to a^+} f(x) = L$ same

Ex: $f(x) = \begin{cases} 3x^2 & \text{if } x < 0 \\ 2x+1 & \text{if } x > 0 \end{cases}$ Find $\lim_{x \to 0} f(x)$

$\lim_{x \to 0^+} 2x+1 = 1$

$\lim_{x \to 0^-} 3x^2 = 0$

$\Big\} \lim_{x \to 0} f(x) = \underline{\underline{\text{d.n.e.}}}$

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Panel 10

$f(x) = \begin{cases} 3x^2-1 & \text{if } x < 1 \\ 5x-3 & \text{if } x > 1 \end{cases}$ $\lim_{x \to 1} f(x) = 2$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} 5x-3 = 2$

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} 3x^2-1 = 2$

$\Big\}$ agree, so

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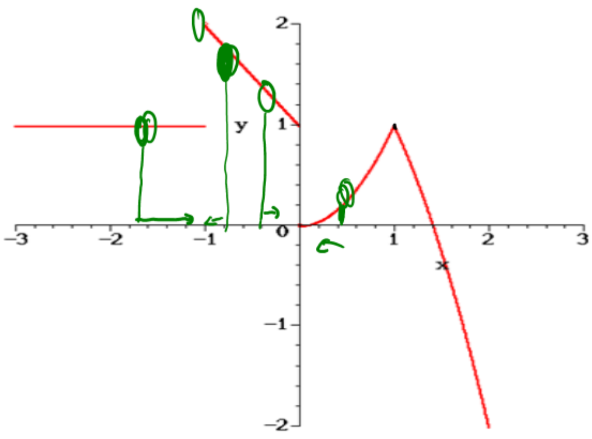
Panel 11

Graphical limits



$\lim_{x \to -1^-} f(x) = 1$

$\lim_{x \to -1^+} f(x) = 2$

$\lim_{x \to 0^-} f(x) = 1$

$\lim_{x \to 0^+} f(x) = 0$

$\lim_{x \to 1^-} f(x) = 1$

$\lim_{x \to 1^+} f(x) = 1$

put a ring on the graph that is attached by a rubber band to x-axis.
Slide towards x

$\lim_{x \to -1} f(x)$ $\lim_{x \to 0} f(x)$ $\lim_{x \to 1} f(x)$

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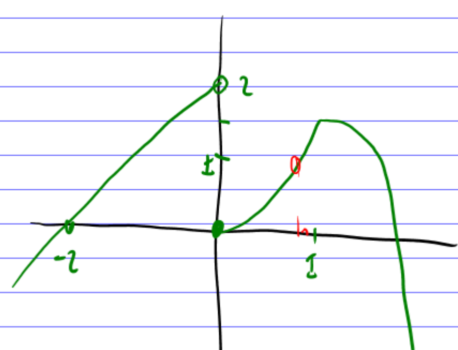
Panel 12



a) $\lim_{x \to -2^-} f(x) = 0$

b) $\lim_{x \to 0^+} f(x) = 0$

c) $\lim_{x \to 1^-} f(x) = 1.5$

d) $\lim_{x \to -2} f(x) = 0$

e) $\lim_{x \to 0} f(x) = \text{d.n.e}$

f) $\lim_{x \to 1} f(x) = 1.5$

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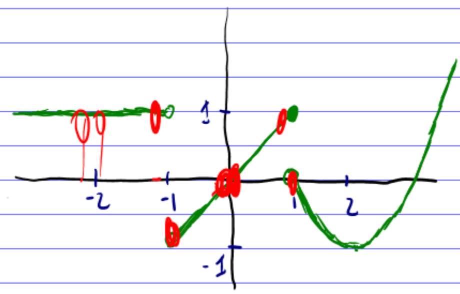
Panel 13

Ex: Consider the graph of the function shown below



a) $\lim_{x \to -2} f(x) = 1$

b) $\lim_{x \to 0^+} f(x) = 0$

c) $\lim_{x \to -1^-} f(x) = 1$

d) $\lim_{x \to 1^+} f(x) = -1$

e) $\lim_{x \to 0} f(x) = 0$

f) $\lim_{x \to -1} f(x)$ d.n.e.

g) $\lim_{x \to 1} f(x) =$ d.n.e.

Quiz on Wed

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