Panel 1

Last Time

Exp function: $y = b^x$ or $y = e^x$, $e \approx 2.71$ Euler's #

Log function: $y = \log_b(x) \Leftrightarrow b^y = x$ $\quad \underbrace{\log(x)}_{\text{common base } 10}$ $\quad \underbrace{\ln(x)}_{\text{nat. base } e}$

Properties of log-functions:

$$\log_b(x) = y \Leftrightarrow b^y = x$$

$b^y = 1$ $\quad (y) = \log_b(1) = 0$ and $\log_b(b) = 1$ $\quad y, \; b^y = b$

$$\log_b(b^x) = x \quad \text{and} \quad b^{\log_b(x)} = x$$

$$\log_b(x^p) = p \cdot \log_b(x)$$

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Panel 2

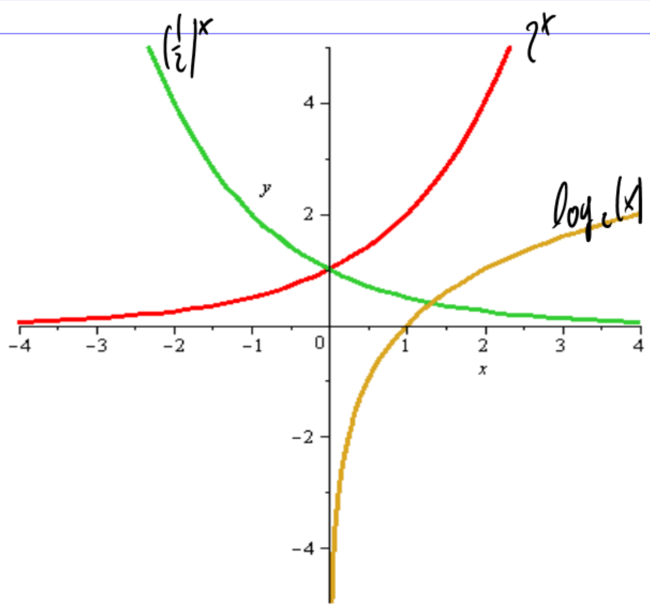
Graphs of exp and log functions

Identify

a) $f(x) = 2^x$

b) $g(x) = \left(\frac{1}{2}\right)^x$

c) $h(x) = \log_2(x)$



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Panel 3

Example: Solve $5 + 3 \cdot 4^{x-1} = 12$

$3 \cdot 4^{x-1} = 7$

$4^{x-1} = \frac{7}{3} \quad | \log_4$

$\log_4(4^{x-1}) = \log_4(\frac{7}{3})$

$(x-1)\log_4(4) = \log_4(\frac{7}{3})$

$x-1 = \log_4(\frac{7}{3})$

$x = \log_4(\frac{7}{3}) + 1$

$4^{x-1} = 7/3 \quad | \ln(\,)$

$\ln(4^{x-1}) = \ln(7/3)$

$(x-1)\ln(4) = \ln(7/3)$

$(x-1)1.386 = 0.8473$

$x-1 = 0.8473/1.386 = 0.611$

$x = 0.611 + 1$

$= 1.611$

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Panel 4

Solve the equation $2^x = 100$ by

a) using $\log_2$: $2^x = 100 \quad | \log_2$

$\log_2(2^x) = \log_2(100)$

$x = \log_2(100) \iff 2^x = 100$

$x \approx 6.?$

b) using $\log$: $2^x = 100 \quad | \log$

$\log(2^x) = \log(100)$

$x \log(2) = \log(100) \Rightarrow x = \frac{\log(100)}{\log(2)} = 6.64$

c) using $\ln(x)$ $2^x = 100 \quad | \ln(\,)$

$\ln(2^x) = \ln(100)$

$x \ln(2) = \ln(100), \quad x = \frac{\ln(100)}{\ln(2)} = 6.64$

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Panel 5

Suppose the number of milligrams of a radioactive substance is $N(t) = 100\, e^{-0.05t}$. How long does it take for ½ of the substance to disappear?

Initial amount: $N(0) = 100$

Want: $N(t) = 50$

$$100\, e^{-0.05t} = 50$$

$$e^{-0.05t} = 1/2 \qquad | \ln(\ )$$

$$\cancel{\ln}(\cancel{e}^{-0.05t}) = \ln(1/2)$$

$$-0.05t = \ln(1/2) \nearrow t = \frac{\ln(1/2)}{-0.05} = \underline{\underline{6.93}}$$

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Panel 6

If you invest \$100 at 10% compounded monthly, when would you have \$1000?

$$A = P(1+r)^t = 100\left(1+\frac{0.1}{12}\right)^{\underset{\text{count}}{12t}} = 1000$$

$$\underline{100}\left(1+\frac{0.1}{12}\right)^{12t} = 1000$$

$$1.0083^{12t} = 10 \qquad |\ln(\ )$$

$$\ln(1.0083^{12t}) = \ln(10)$$

$$12t \cdot \ln(1.0083) = \ln(10) \Rightarrow t = \frac{\ln(10)}{\ln(1.0083)\, 12} = 23.21$$

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Panel 7

$\log_3(\sqrt[7]{3}) = y$, $3^y = \sqrt[7]{3} = 3^{1/7}$, $y = 1/7$

$\log_x(12-x) = 2$ $x^2 = 12-x$

$x^2+x-12=0$

$(x+4)(x-3)=0,\quad x=-4$ or 3

$\log_{-4}$ is not defined.

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Panel 8

$f(x) = 2x^2-3.$ $\frac{f(x+h)-f(x)}{h}$ $f(\quad) = 2(\quad)^2 - 3$

foil

$f(x+h) = 2(x+h)^2 - 3 = 2(x^2+2xh+h^2) - 3 = 2x^2+4xh+2h^2-3$

$\frac{f(x+h)-f(x)}{h} = \frac{(\cancel{2x^2}+4xh+2h^2\cancel{-3})-(\cancel{2x^2}\cancel{-3})}{h} = \frac{4xh+2h^2}{h} = \frac{\cancel{h}(4x+2h)}{\cancel{h}} = 4x+2h$

$f(x) = 3xe^{x^2}$ $f(-x) = \boxed{-3xe^{x^2}} = -f(x)$

$f(-x) = -f(x)$ odd

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Panel 9

$\log_3\left(\frac{1}{81}\right) = y$ $\quad 3^y = \frac{1}{81} = 81^{-1} = (3^4)^{-1} = 3^{-4}$ $\quad y = -4$

$\log(6x-2) = 2$

$10^2 = 6x-2$

$100 = 6x-2$

$102 = 6x$

$\frac{102}{6} = x$

$17 = x$

$3^{4x} = 9^{x+1}$

$3^{4x} = (3^2)^{x+1}$

$3^{4x} = 3^{2x+2}$ $\quad 4x = 2x+2$

$2x = 2, x = 1$

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