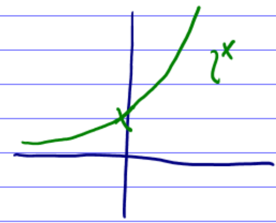
Panel 1

<u>Last Time</u>



Exp. function, Properties.

Range: $(0,\infty)$

Domain: $(-\infty,\infty)$

$5^0=1$

$2^x - 1$

Compound Interest:

$A = P(1+r)^t$, r = rate <u>per period</u>, t = # of <u>periods</u>

1

Panel 2

$P = P_0(1-r)^t$, $P_0 = 350000$, $r = 0.015$

$P = 350000(1-0.015)^3 = \underline{334485}.\cancel{06875}$

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Panel 3

Thought Experiment: Invest \$1 at 100% for one year, compounded n-times per year:

$$S = 1\cdot\left(1+\frac{1}{n}\right)^n = \left(1+\frac{1}{n}\right)^n$$

n	S
1	2
10	2.593
100	2.7048
1000	2.7169
10000	2.7181
100 000	2.7181

This eventually stabilizes to a number 2.71817...

That number is called

e (Euler's number)

(strange number)

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Panel 4

The Natural Exponential Function

Define the number $e = 2.7182\ldots$ (Euler's Number)

Then $f(x) = e^x$ is natural exp. function

Ex: $f(0) = 1$, $f(1) = e = 2.7181\ldots$ 2^x

$f(3) = 20.08$

$f(-2) = e^{-2} = 1/e^2 = \underline{0.1353}$

$f(1/3) = e^{1/3} = \sqrt[3]{e} = \underline{1.3955}$

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Panel 5

Population Growth: The projected population P of a city is $P(t) = 100000\, e^{0.05t}$

where t is the number of years after 1990.

a) What was the population in 1990

$P(0) = 100000$

b) Predict population in 2020

$P(30) = 100000\, e^{0.05 \cdot 30} = 448000$

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Panel 6

Radioactive Decay: A radioactive element decays such that after t days the amount left (in mg) is

$N = 100\, e^{-0.062t}$, t in days

a) How many mg are initially present?

$N(0) = 100$ mg

b) How much after 10 days?

$N(10) = 100 e^{-0.062 \cdot 10} = 100 e^{-0.62} = 53.04$

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Panel 7

Logarithm Functions:

$\sqrt{x^2} = x$

$(\sqrt{x})^2 = x$

Recall: $y = x^2$ is same as $\sqrt{y} = x$

That means $\sqrt{y}$ is that number x s.t. $x^2 = y$. $\sqrt{9} = 3$

Define: $y = \log_b(x)$ is that number y s.t. $b^y = x$

"log base b of x"

Note: $5^{\log_5(x)} = x$, $\log_5(5^x) = x$

Ex: $\log_2(8) = y$ is that number s.t. $2^y = 8$, so $y = 3$

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Panel 8

Some examples:

Find the following values:

$y = \log_5(25)$ $\quad 5^y = 25$, $y = 2$

$y = \log_3(81)$ $\quad 3^y = 81 =$, $y = 4$

$y = \log_{16}(1)$ $\quad 16^y = 1$, $y = 0$

$y = \log_2(\frac{1}{16})$ $\quad 2^y = 1/16 = 2^{-4}$, $y = -4$

$y = \log_{64}(8)$ $\quad 64^y = 8$, $y = 1/2$

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Panel 9

Graph $y = \log_2(x)$ and $y = 2^x$ in one system

x	$\log_2(x) = y$	alternate
1	0	$2^y = 1$
2	1	$2^y = 2$
3	—	$2^y = 3$
4	2	$2^y = 4$
8	3	
$\frac{1}{2}$	-1	$2^y = 1/2$
$\frac{1}{4}$	-2	
$\frac{1}{8}$	-3	

The graphs of 2^x and $\log_2(x)$ are reflection about the main diagonal

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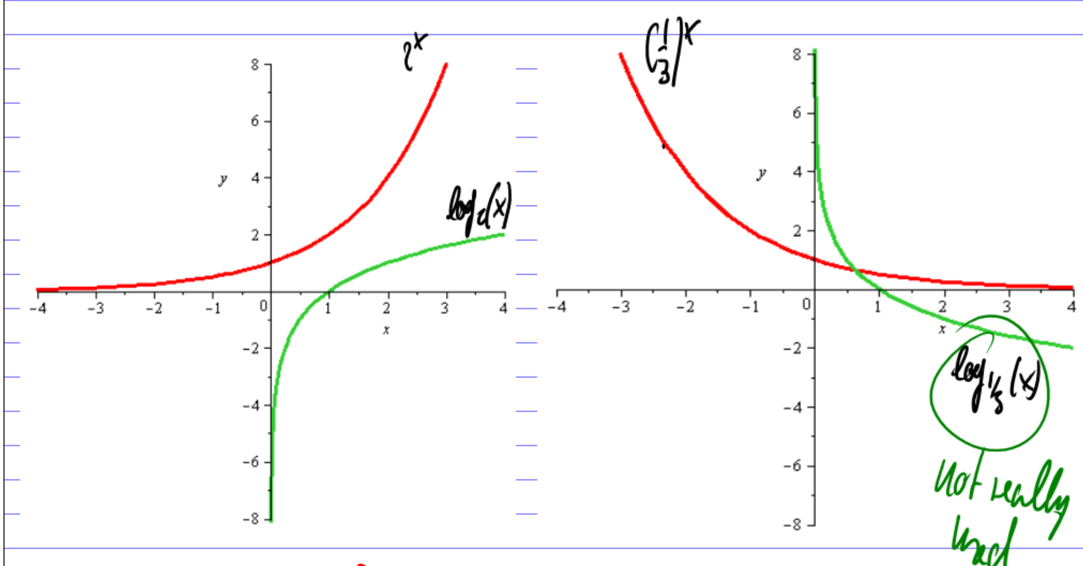
Panel 10

Graphs of 5^x and $\log_5$



$y = \log_5(x)$ and $y = 5^x$ are opposites, their graphs are reflected about main diagonal.

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Panel 11

Solve the following equations:

a) $\log_2(x) = 4$ $2^4 = x = 16$

b) $\log_3(x+1) = 7$ $3^7 = x+1$, $x = 3^7 - 1 = 2187 - 1 = \underline{\underline{2186}}$

c) $\log_x(49) = 2$ $x^2 = 49$, $x = 7$

d) $2^{5x} = 4 = 2^2$, $2^{5x} = 2^2 \Rightarrow 5x = 2$, $\underline{\underline{x = \frac{2}{5}}}$

Panel 12

<u>Special Logarithms</u>

$\log_b(x) = y \iff b^y = x$

Natural logarithm:

$y = \log_e(x) = \ln(x)$

Common logarithm:

$y = \log_{10}(x) \Rightarrow y = \log(x)$

Panel 13

Properties of Logarithms

$\log_5(1) = y = 0$, $5^y = 1$, $y =$

$\log_5(5) = 1$ y = $5^y = 5$

domain of $\log_5(x)$: $(0, \infty)$

range of $\log_5(x)$: $(-\infty, \infty)$

$\left(\begin{array}{l} \exp: (-\infty, \infty) \\ = (0, \infty) \end{array} \right)$

$\log_5(5^x) = x$

$5^{\log_5(x)} = x$

$\log_5(x^p) = p \log_5(x)$

$\left(\log_4(x y) = \log_5(x) + \log_5(y) \right)$

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Panel 14

Examples

Solve $25^{x+2} = 5^{3x-4}$

Solve $5^{2x-1} = 125$ and $5^{2x-1} = 100$

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Panel 15

Example: Solve $5 + 3 \cdot 4^{x-1} = 12$

$$3 \cdot 4^{x-1} = 7$$

$$4^{x-1} = 7/3 \qquad \Big| \log_4(\,)$$

$$\cancel{\log_4}(\cancel{4}^{x-1}) = \log_4\left(\tfrac{7}{3}\right)$$

$$x - 1 = \log_4\left(\tfrac{7}{3}\right)$$

$$x = \log_4\left(\tfrac{7}{3}\right) + 1$$

stuck!

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