Panel 1

Last Time

- Linear functions and their graphs

$m = \frac{y_2 - y_1}{x_2 - x_1}$ slope    $ax + by + c = 0$

$y = mx + b$

$y - y_1 = m(x - x_1)$

x-intercept: set $y = 0$  hard, several, $f(x) = x^2 - 7x + 9 = 0$

y-intercept set $x = 0$  easy

parallel

perp.

demand and supply functions
down up

systems of linear equations - substitution method

Panel 2

Ex: Are these lines (A) Parallel [✓ (B) Perpend.] (C) Neither

$y - 5 = 2x - 2$    $y = 2x - 2 + 5 = 2x + 3$, $m = 2$

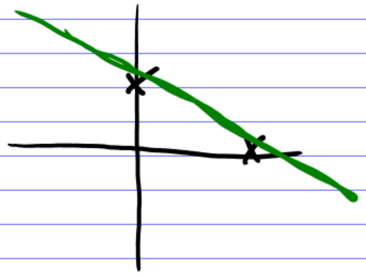
$6y + 3x = 5$    $6y = 5 - 3x$

$y = \frac{5}{6} - \frac{3}{6}x = \frac{5}{6} - \frac{1}{2}x$    $m = -1/2$

Ex: Graph $2x + 3y - 6 = 0$

x-int: set $y = 0$, $x = 3$

y-int: set $x = 0$, $y = 2$

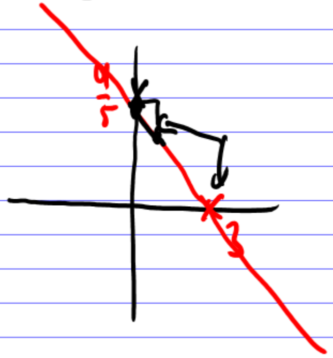


Panel 3

$3x + 5y - 9 = 0$

$5y = -3x + 9$

$y = -\frac{3}{5}x + \underline{\frac{9}{5}}$



Panel 4

Ⓑ Substitution Method: solve one equation for one variable and substitute result into other equation.

(for systems of equations)

Ex: $x - 2y = 8$ $\qquad x = 8 + 2y$

$2x + 4y = 0$ $\qquad 2(8 + 2y) + 4y = 0$

$16 + 4y + 4y = 0$

$16 + 8y = 0, \quad \underline{\underline{y = -2}}$

$\underline{\underline{x = 4}}$

Panel 5

Solve by elimination or substitution?

$2x + 7y = 3 \quad \mid -2$

$4x + 15y = 9$

$-\cancel{4x} - 14y = -6$

$\cancel{4x} + 15y = 9$

$\boxed{y = 3}$

$x = -9$

$2x + 7 \cdot 3 = 3$

$2x + 21 = 3$

$2x = -18$

$x = 6$

$\boxed{x = 3y}$

$2x + 4y = 20$

Subst

$2(3y) + 4y = 20 \quad 6y + 4y = 20 \Rightarrow y = 2$

$x + 5y = 2$

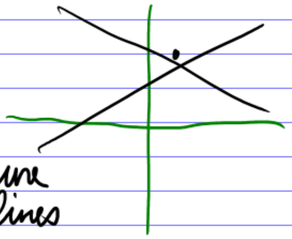
$\frac{1}{2}x = 1 - \frac{5}{2}y \quad \mid \cdot 2$

$\boxed{x} = 2 - 5y$

$(2-5y) + 5y = 2$

$2 = 2$

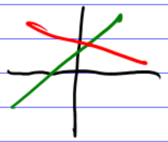
always true because same lines



Panel 6

When solving a system of linear equations, it could have:

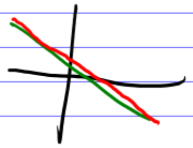
a) one solution



b) no solution

(false equation)

c) infinitely many solution



(true equ)

Panel 7

Application: Suppose for product X we have

$$p = \underbrace{-\frac{1}{180} q + 12}_{} \quad \text{(demand)}$$

$$\boxed{p} = \frac{1}{300} q + 8 \quad \text{(supply)}$$

Def. The point where demand is equal to the supply is called equilibrium point (both producers + consumers are happy)

$$-\frac{1}{180} q + 12 = \frac{1}{300} q + 8$$

$$4 = \frac{1}{300} q + \frac{1}{180} q$$

$$4 = \left(\frac{1}{300} + \frac{1}{180}\right) q = 0.008888.. q,$$

$$q = \frac{4}{0.00888..} = \boxed{450}$$

$$p = 9.5$$

Panel 8

Final application: A chemical manufacturer wants to produce 500 liters of a 25% acid solution. She has solutions of 30% and 18% acidity in stock. How should they be mixed? How many liters from each?

Mixing problems

$x$ = # of liters of 30%

$y$ = # of liters of 18%

Want: $x + y = 500$ $\qquad$ $x + y = 500$

$x \cdot 0.3 + y \cdot 0.18 = 500 \cdot 0.25$ $\qquad$ $0.3x + 0.18y = 125$

Rest is $\boxed{\text{HW}}$

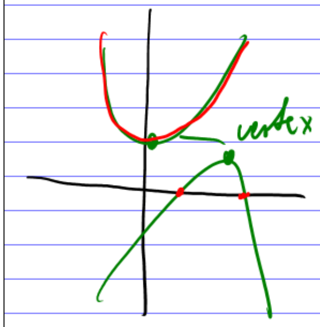
Panel 9

## Quadratic Functions

- $\boxed{y = ax^2 + bx + c}$, $a \neq 0$
- Graph is parabola which
  opens up : if $a > 0$
  Opens down: if $a < 0$
- vertex: lowest/highest point
  $x = -b/2a$, $y = f(-b/2a)$
- y-intercept: set $x = 0$, $\Rightarrow \underline{y = c}$
- x-intercept:

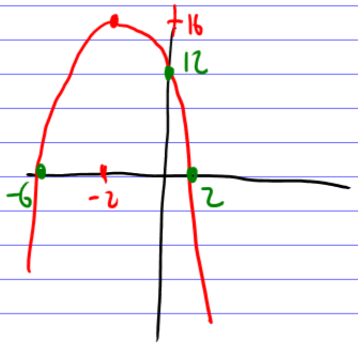
$$x_{1/2} = \underline{\underline{\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}}$$

two answers



Panel 10

$\underline{\underline{\text{Ex:}}}$ Graph $f(x) = \underline{-x^2 - 4x + 12}$



down

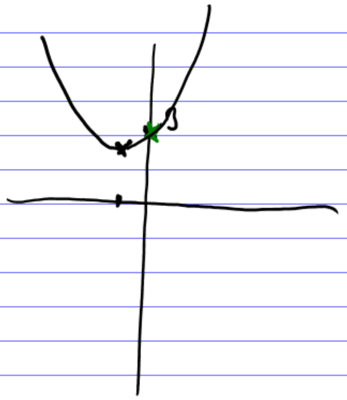
vertex: $x = -\frac{b}{2a} = -\frac{-4}{-2} = \boxed{\underline{\underline{-2}}}$

$y = -(-2)^2 - 4(-2) + 12 =$
$= -4 + 8 + 12 = 16$

$0 = -x^2 - 4x + 12 \quad | \cdot -1$
$= x^2 + 4x - 12$
$= (x+6)(x-2)$
$x = 2$, or $-6$

Panel 11

Ex: Graph $f(x) = 2x^2 + 2x + 3$ and find range



y-int: $y = 3$

vertex: $x = -\frac{b}{2a} = -\frac{2}{4} = \underline{-\frac{1}{2}}$

$y = 2\left(-\frac{1}{2}\right)^2 + 2\left(-\frac{1}{2}\right) + 3 =$

$= \frac{1}{2} - 1 + 3 = \underline{2.5}$

x-int: $y = \boxed{0 = 2x^2 + 2x + 3}$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-2 \pm \sqrt{4 - 24}}{4}$ no solution

Panel 12

Ex: Suppose the demand for a product is
$p = 1000 - 2q$, $q$ = # of units and $p$ price
per unit if $q$ units are demanded (weekly).
Find max revenue.

$R(q) = p \cdot q = (1000 - 2q) \cdot q = 1000q - 2q^2$

is a parabola going down, so max occurs at vertex.

$q = -\frac{b}{2a} = -\frac{1000}{-4} = \frac{1000}{4} = \underline{250}$

Important

Sell $\underline{250 \text{ units}}$ for max. profit of $\underline{R(250)} = \#$