

106 Chapter 2 Functions and Graphs

Since the distinct input values -4 and 4 produce the same output, the function is not one-to-one. Looking at it another way, we have the following general rule, called the **horizontal-line test**. If a horizontal line L can be drawn that intersects the graph of a function in at least two points, then the function is *not* one-to-one. When no such horizontal line can be drawn, the function is one-to-one.

PROBLEMS 2.5

In Problems 1 and 2, locate and label each of the points, and give the quadrant, if possible, in which each point lies.

1. $(-2, -5), (3, -1), (-\frac{1}{3}, 4), (1, 0)$

2. $(-4, 5), (3, 0), (1, 1), (0, -6)$

3. Figure 2.23(a) shows the graph of $y = f(x)$.

- (a) Estimate $f(0), f(2), f(4)$, and $f(-2)$.
- (b) What is the domain of f ?
- (c) What is the range of f ?
- (d) What is an x -intercept of f ?

4. Figure 2.23(b) shows the graph of $y = f(x)$.

- (a) Estimate $f(0)$ and $f(2)$.
- (b) What is the domain of f ?
- (c) What is the range of f ?
- (d) What is an x -intercept of f ?

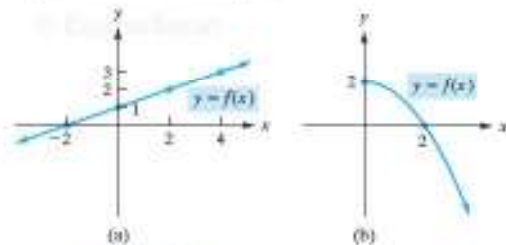


FIGURE 2.23 Diagram for Problems 3 and 4.

5. Figure 2.24(a) shows the graph of $y = f(x)$.

- (a) Estimate $f(0), f(1)$, and $f(-1)$.
- (b) What is the domain of f ?
- (c) What is the range of f ?
- (d) What is an x -intercept of f ?

6. Figure 2.24(b) shows the graph of $y = f(x)$.

- (a) Estimate $f(0), f(2), f(3)$, and $f(4)$.
- (b) What is the domain of f ?
- (c) What is the range of f ?
- (d) What is an x -intercept of f ?



In Problems 7–20, determine the intercepts of the graph of each equation, and sketch the graph. Based on your graph, is y a function of x , and, if so, is it one-to-one and what are the domain and range?

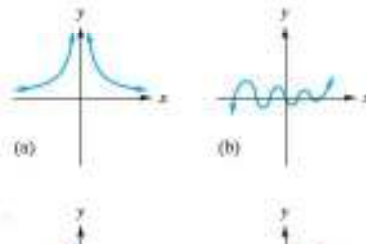
- | | |
|----------------------|-------------------------|
| 7. $y = 2x$ | 8. $y = -x + 1$ |
| 9. $y = 3x - 5$ | 10. $y = 3 - 2x$ |
| 11. $y = x^2 + x$ | 12. $y = \frac{2}{x^2}$ |
| 13. $x = 0$ | 14. $y = 4x^2 - 16$ |
| 15. $y = x^3$ | 16. $x = 3$ |
| 17. $x = - y $ | 18. $x^2 = y^2$ |
| 19. $2x + y - 2 = 0$ | 20. $x + y = 1$ |

In Problems 21–34, graph each function and give the domain and range. Also, determine the intercepts.

- | | |
|---------------------------------|----------------------------------|
| 21. $u = f(v) = 2 + v^2$ | 22. $f(x) = 5 - 2x^2$ |
| 23. $y = h(x) = 3$ | 24. $g(x) = -17$ |
| 25. $y = h(x) = x^2 - 4x + 1$ | 26. $y = f(x) = -x^2 + x + 6$ |
| 27. $f(t) = -t^3$ | 28. $p = h(q) = 1 + 2q + q^2$ |
| 29. $x = f(t) = \sqrt{t^2 - 9}$ | 30. $F(r) = -\frac{1}{r}$ |
| 31. $f(x) = 3x + 2 $ | 32. $v = H(u) = u - 3 $ |
| 33. $F(t) = \frac{16}{t^2}$ | 34. $y = f(x) = \frac{2}{x - 4}$ |

In Problems 35–38, graph each case-defined function and give the domain and range.

35. $c = g(p) = \begin{cases} p + 1 & \text{if } 0 \leq p < 7 \\ 5 & \text{if } p \geq 7 \end{cases}$
36. $y(x) = \begin{cases} 3x & \text{if } 0 \leq x < 2 \\ 10 - x^2 & \text{if } x \geq 2 \end{cases}$
37. $g(x) = \begin{cases} x + 6 & \text{if } x \geq 3 \\ x^2 & \text{if } x < 3 \end{cases}$ 38. $f(x) = \begin{cases} x + 1 & \text{if } 0 < x \leq 3 \\ 4 & \text{if } 3 < x \leq 5 \\ x - 1 & \text{if } x > 5 \end{cases}$
39. Which of the graphs in Figure 2.25 represent functions of x ?



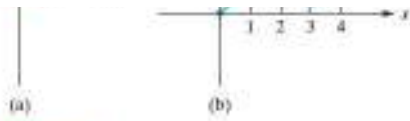


FIGURE 2.24 Diagram for Problems 5 and 6.

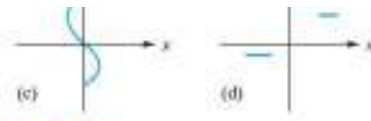


FIGURE 2.25 Diagram for Problem 39.