EXAMPLE 7 **Genetics**

Factorials occur frequently in probability theory.

Suppose two black guinea pigs are bred and produce exactly five offspring. Under certain conditions, it can be shown that the probability P that exactly r of the offspring will be brown and the others black is a function of r, $P = P(r)$, where

$$P(r) = \frac{5!\left(\frac{1}{4}\right)^r\left(\frac{3}{4}\right)^{5-r}}{r!(5-r)!} \qquad r = 0, 1, 2, \ldots, 5$$

The letter P in $P = P(r)$ is used in two ways. On the right side, P represents the function rule. On the left side, P represents the dependent variable. The domain of P is all integers from 0 to 5, inclusive. Find the probability that exactly three guinea pigs will be brown.

Solution We want to find $P(3)$. We have

$$P(3) = \frac{5!\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^2}{3!2!} = \frac{120\left(\frac{1}{64}\right)\left(\frac{9}{16}\right)}{6(2)} = \frac{45}{512}$$

Now Work Problem 35 ◁

PROBLEMS 2.2

In Problems 1–4, determine whether the given function is a polynomial function.

1. $f(x) = x^3 - x^4 + 4$
2. $f(x) = \dfrac{x^2 + 7x - 3}{3}$
3. $g(x) = \dfrac{1}{x^2 + 2x + 1}$
4. $g(x) = 2^{-5}x^3$

In Problems 5–8, determine whether the given function is a rational function.

5. $f(x) = \dfrac{x^2 + x}{x^3 + 4}$
6. $f(x) = \dfrac{3}{2x + 1}$
7. $g(x) = \begin{cases} 1 & \text{if } x < 5 \\ 4 & \text{if } x \geq 5 \end{cases}$
8. $g(x) = 4x^{-4}$

In Problems 9–12, find the domain of each function.

9. $k(z) = 26$
10. $f(x) = \sqrt{\pi}$
11. $f(x) = \begin{cases} 5x & \text{if } x > 1 \\ 4 & \text{if } x \leq 1 \end{cases}$
12. $f(x) = \begin{cases} 4 & \text{if } x = 3 \\ x^2 & \text{if } 1 \leq x < 3 \end{cases}$

In Problems 13–16, state (a) the degree and (b) the leading coefficient of the given polynomial function.

13. $F(x) = 7x^5 - 2x^2 + 6$
14. $g(x) = 9x^2 + 2x + 1$
15. $f(x) = \dfrac{1}{\pi} - 3x^3 + 2x^8 + x^7$
16. $f(x) = 9$

In Problems 17–22, find the function values for each function.

17. $f(x) = 8$; $f(2)$, $f(t + 8)$, $f(-\sqrt{17})$
18. $g(x) = |x - 3|$; $g(10)$, $g(3)$, $g(-3)$
19. $F(t) = \begin{cases} 2 & \text{if } t > 1 \\ 0 & \text{if } t = 1 \end{cases}$;
21. $G(x) = \begin{cases} x - 1 & \text{if } x \geq 3 \\ 3 - x^2 & \text{if } x < 3 \end{cases}$;
$G(8)$, $G(3)$, $G(-1)$, $G(1)$
22. $F(\theta) = \begin{cases} 2\theta - 5 & \text{if } \theta < 2 \\ \theta^2 - 3\theta + 1 & \text{if } \theta > 2 \end{cases}$;
$F(3)$, $F(-3)$, $F(2)$

In Problems 23–28, determine the value of each expression.

23. $6!$
24. $(3 - 3)!$
25. $(4 - 2)!$
26. $6! \cdot 2!$
27. $\dfrac{n!}{(n-1)!}$
28. $\dfrac{8!}{5!(8-5)!}$

29. **Subway Ride** A return subway ride ticket within the city costs $2.50. Write the cost of a return ticket as a function of a passenger's income. What kind of function is this?

30. **Geometry** A rectangular prism has length three more than its width and height one less than twice the width. Write the volume of the rectangular prism as a function of the width. What kind of function is this?

31. **Cost Function** In manufacturing a component for a machine, the initial cost of a die is $850 and all other additional costs are $3 per unit produced. (a) Express the total cost C (in dollars) as a linear function of the number q of units produced. (b) How many units are produced if the total cost is $1600?

32. **Investment** If a principal of P dollars is invested at a simple annual interest rate of r for t years, express the total accumulated amount of the principal and interest as a function of t. Is your result a linear function of t?

33. **Sales** To encourage large group sales, a theater charges two rates. If your group is less than 12, each ticket costs $9.50. If your group is 12 or more, each ticket costs $8.75. Write a piece-defined

$F(12), F(-\sqrt{3}), F(1), F\left(\frac{18}{5}\right)$

20. $f(x) = \begin{cases} 4 & \text{if } x \geq 0 \\ 3 & \text{if } x < 0 \end{cases}$;
$f(3), f(-4), f(0)$

function to represent the cost of buying n tickets.

34. Factorials The business mathematics class has elected a grievance committee of five to complain to the faculty about the introduction of factorial notation into the course. They decide that they will be more effective if they label themselves as members A, G, M, N, and S, where member A will lobby faculty with