86   Chapter 2 Functions and Graphs

APPLY IT ▶

4. For the supply function given by the following table, determine the weekly revenue function, assuming that all units supplied are sold.

p Price per Unit in Dollars	q Quantity Supplied per Week
500	11
600	14
700	17
800	20

EXAMPLE 6   Supply Schedule

The table in Apply It 4 on this page is a *supply schedule*. Such a table lists for each of certain prices p of a certain product the quantity q that producers will supply per week at that price. For each price, the table provides exactly one quantity so that it exhibits q as a function of p.

But also, for each quantity, the table provides exactly one price so that it also exhibits p as a function of q. If we write $q = f(p)$, then the table provides

$$f(500) = 11 \quad f(600) = 14 \quad f(700) = 17 \quad f(800) = 20$$

If we write $p = g(q)$, then the table also provides

$$g(11) = 500 \quad g(14) = 600 \quad g(17) = 700 \quad g(20) = 800$$

Observe that we have $g(f(p)) = p$, for all p, and $f(g(q)) = q$, for all q. We will have more to say about pairs of functions of this kind in Section 2.4. Both functions determined by this table are called **supply functions**.

Now Work Problem 53 ◁

PROBLEMS 2.1

In Problems 1–4, determine whether the given functions are equal.

1. $f(x) = \sqrt{x^2}; \ g(x) = x$

2. $G(x) = (\sqrt{x+1})^2; \ H(x) = x + 1$

3. $h(x) = \dfrac{|x|}{x}; \ k(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$

4. $f(x) = \begin{cases} \dfrac{x^2 - 4x + 3}{x - 3} & \text{if } x \neq 3 \\ 2 & \text{if } x = 3 \end{cases}$;
$g(x) = x - 1$

In Problems 5–16, give the domain of each function.

5. $f(x) = \dfrac{6}{x-1}$ **6.** $g(x) = \dfrac{x}{5}$

7. $h(x) = \sqrt{x-3}$ **8.** $K(z) = \dfrac{1}{\sqrt{z-1}}$

9. $f(z) = 3z^2 + 2z - 4$ **10.** $H(x) = \dfrac{x^2}{x+3}$

11. $f(x) = \dfrac{9x-9}{2x+7}$ **12.** $g(x) = \sqrt{4x+3}$

13. $g(y) = \dfrac{4}{y^2 - 4y + 4}$ **14.** $\phi(x) = \dfrac{x+5}{x^2+x-6}$

15. $h(x) = \dfrac{3-x^2}{3x^2 - 5x - 2}$ **16.** $G(r) = \dfrac{2}{r^2+1}$

In Problems 17–28, find the function values for each function.

17. $f(x) = 2x + 1; f(0), f(3), f(-4)$

18. $H(x) = 5x^2 - 3; H(4), H(\sqrt{2}), H\left(\dfrac{2}{3}\right)$

19. $G(x) = 2 - x^2; G(-8), G(u), G(u^2)$

20. $F(x) = -7x + 1; F(s), F(t+1), F(x+3)$

21. $\gamma(u) = 2u^2 - u; \gamma(-2), \gamma(2v), \gamma(x+a)$

25. $k(x) = \dfrac{x-5}{x^2+1}; k(5), k(2x), k(x+h)$

26. $k(x) = \sqrt{x-3}; k(4), k(3), k(x+1) - k(x)$

27. $f(x) = x^{4/3}; f(0), f(64), f\left(\dfrac{1}{8}\right)$

28. $g(x) = x^{2/3}; g(32), g(-64), g(t^{10})$

In Problems 29–36, find (a) $f(x+h)$ and (b) $\dfrac{f(x+h) - f(x)}{h}$; simplify your answers.

29. $f(x) = 4x - 5$ **30.** $f(x) = \dfrac{x}{3}$

31. $f(x) = x^2 + 2x$ **32.** $f(x) = 3x^2 - 2x - 1$

33. $f(x) = 3 - 2x + 4x^2$ **34.** $f(x) = x^3$

35. $f(x) = \dfrac{1}{x-1}$ **36.** $f(x) = \dfrac{x+8}{x}$

37. If $f(x) = 5x + 3$, find $\dfrac{f(3+h) - f(3)}{h}$.

38. If $f(x) = 2x^2 - x + 1$, find $\dfrac{f(x) - f(2)}{x-2}$.

In Problems 39–42, is y a function of x? Is x a function of y?

39. $9y - 3x - 4 = 0$ **40.** $x^4 - 1 + y = 0$

41. $y = 7x^2$ **42.** $x^2 + y^2 = 1$

43. The formula for the area of a circle of radius r is $A = \pi r^2$. Is the area a function of the radius?

44. Suppose $f(b) = a^2 b^3 + a^3 b^2$. (a) Find $f(a)$. (b) Find $f(ab)$.

45. Value of Business A business with an original capital of $50,000 has income and expenses each week of $7200 and $4900,

22. $h(v) = \dfrac{1}{\sqrt{v}}$; $h(16)$, $h\left(\dfrac{1}{4}\right)$, $h(1-x)$

23. $f(x) = x^2 + 2x + 1$; $f(1)$, $f(-1)$, $f(x+h)$

24. $H(x) = (x+4)^2$; $H(0)$, $H(2)$, $H(t-4)$

respectively. If all profits are retained in the business, express the value V of the business at the end of t weeks as a function of t.

46. Depreciation If a $30,000 machine depreciates 2% of its original value each year, find a function f that expresses the machine's value V after t years have elapsed.