Panel 1

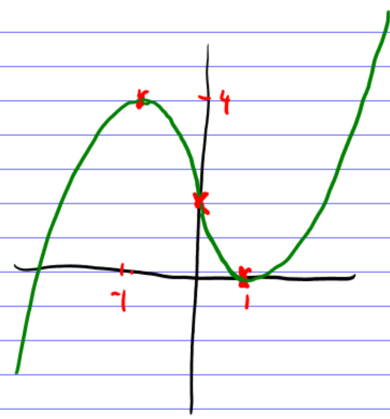
#6) Graph $f(x) = x^3 - 3x + 2$ and find all extrema etc.

$f'(x) = 3x^2 - 3 = 0$

$3(x^2-1) = 0 \Rightarrow x = \pm 1$ are critical points

$f''(x) = 6x = 0 \Rightarrow x = 0$ is possible infl. point

	-1	0	1	2
f'	+	−	−	+
f''	−	−	+	+
f				



$f(0) = 2$

$f(1) = 0$

$f(-1) = 4$

1

Panel 2

find $f(x)$ if $f'(x) = \sqrt{x} - 3$ and $f(4) = -1$.

$f(x) = \int \sqrt{x} - 3\, dx = \int x^{1/2} - 3\, dx = \frac{2}{3} x^{3/2} - 3x + C$

$-1 = f(4) = \frac{2}{3}(4)^{3/2} + 3 \cdot 4 + C = \frac{16}{3} + 12 + C$

$-1 = 15\frac{1}{3} + C$

$-1 - 15\frac{1}{3} = C$

$-16\frac{1}{3} = C$

$$\boxed{f(x) = \frac{2}{3} x^{3/2} - 3x - 16\frac{1}{3}}$$

2

Panel 3

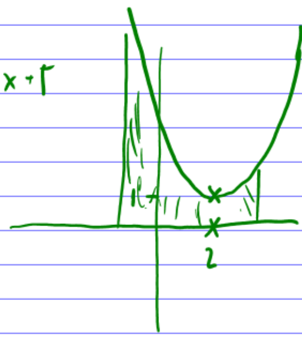
Find the area under the curve $y = x^2 - 4x + 5$ from x=-1 to x=3. Sketch the region.

$\Rightarrow \quad A = \int_{-1}^{3} x^2 - 4x + 5 \, dx = \frac{1}{3}x^3 - 4\,\frac{1}{2}x^2 + 5x \Big|_{-1}^{3}$

$\left(\frac{1}{3}(27) - 18 + 15\right) - \left(-\frac{1}{3} - 2 - 5\right)$

$9 - 18 + 15 + \frac{1}{3} + 2 + 5 = 13\frac{1}{3} = \frac{40}{3}$

$x^2 - 4x + 5$



3

Panel 4

0. If $R(x) = 60x - .3x^2$ and $C(x) = 12x + 2$, find the maximum profit and the number of units that must be produced and sold to maximize the profit.

$P(x) = R(x) - C(x) = 60x - 0.3x^2 - 12x - 2 = -0.3x^2 + 48x - 2$

$P'(x) = -0.6x + 48 = 0$

$\Rightarrow \quad x = \frac{48}{0.6} = 80$ # of units for max. profit

Max. profit is: $P(80) = \#$

4

Panel 5

1. Suppose \$8000 is invested in an account. How much money is in the account in 6 years if the interest rate is 5% compounded: a) monthly b) continuously?

a) $S = P(1+r)^n = 8000\left(1+\frac{0.05}{12}\right)^{6\cdot 12}$

b) $S = Pe^{rt} = 8000\left(e^{0.05\cdot 6}\right)$

Remember cont. compounding

5

Panel 6

An object is dropped from a certain height. It is known that it will fall a distance of $s(t) = 16t^2$ where s is in feet and t is in seconds. What is the average speed from 3 to 5 seconds. "rate of change"

~~Say s(3) = 10 and s(5) = 20~~

$\frac{s(5) - s(3)}{5-3} = \frac{16\cdot 25 - 16\cdot 9}{2} = \frac{16(25-9)}{2} = 8\cdot 16 = \underline{128}$

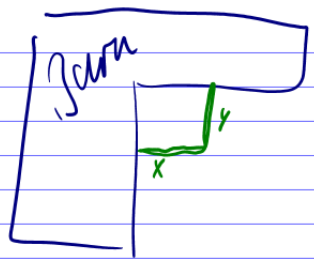
Contrast with instant. rate of change at $t=4$, i.e. velocity

$s'(t) = 32t$, $s'(4) = 32\cdot 4 = \underline{128}$

6

Panel 7

A rancher has 50 feet of fencing to fence off a rectangular animal pen in the corner of a barn. The corner of the barn will not be fenced. What dimensions of the rectangle will maximize the area?



$50 = x + y \Rightarrow y = 50 - x$

$A = xy$ is max. if: $x = 25, y = 25$

$= x(50-x) = 50x - x^2$

$A'(x) = -2x + 50 = 0$, $\underline{x = 25}$ is max for upside down parabola

7

Panel 8

After t hours of operation, a coal mine is producing coal at a rate of $40 + 2t - 9t^2$ tons of coal per hour. Find the formula for the output of the coal mine after t hours of operation if we know that after 2 hours, 80 tons have been mined.

$f(t)$ is the output function

$f'(t) = 40 + 2t - 9t^2$

$f(t) = \int 40 + 2t - 9t^2 \, dt = \underline{40t - t^2 - 3t^3 + C}$

$\underline{80 = f(2)} = 40 \cdot 2 - (2)^2 - 3(2)^3 = \underline{80 - 4 - 24} = \underline{52}$

$f(t) = 40t - t^2 - 3t^3 + 52$

8

Panel 9

A firm estimates that it will sell N units of a product after spending x dollars on advertising, where $N(x) = -x^2 + 300x + 6$ and x is measured in thousands of dollars. What is the rate of change of the number of units sold with respect to the amount spent on advertising after spending $10 thousand?

$N'(x) = -2x + 300$ at $x = 10$

$-20 + 300 = \underline{280}$ which means N is increasing

at $\underline{x = 280}$

9