

Panel 1

Last Topic: Solving Systems of Equations

Want to solve

$$\begin{aligned} (1) \quad & 3x - y = 1 \\ (2) \quad & x + 2y = 5 \end{aligned}$$

Substitution Method:

solve (1) for  $y$  and  
plug into (2)

Elimination Method

Mult. (1) by 2 and  
add to (2), replace (2) with answer

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Panel 2

Matrix Method

$$\begin{aligned} 3x - y &= 1 \\ x + 2y &= 5 \end{aligned}$$

$$\begin{array}{cc|c} x & y & \\ \hline 3 & -1 & \\ 1 & 2 & \end{array} \begin{array}{l} 2 \times 2 \text{ matrix} \\ \text{rows} \\ \text{cols} \end{array}$$

Definition: (Coefficient Matrix)

the coefficients in front of variables arranged in  
rows + columns (aka table)

Def: (Augmented coefficient matrix)

coefficient matrix + extra column for the  
constant values on the right

$$\left( \begin{array}{cc|c} 3 & -1 & 1 \\ 1 & 2 & 5 \end{array} \right) \begin{array}{l} 2 \times 3 \text{ matrix} \end{array}$$

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Panel 3

Ex: Find the augmented coefficient matrix for the system:

$$\begin{array}{r} 2x + y - z = 3 \\ -2x + 2y + 3z = 8 \\ x + y + z = 1 \end{array} \quad \left( \begin{array}{ccc|c} 2 & 1 & -1 & 3 \\ -2 & 2 & 3 & 8 \\ 1 & 1 & 1 & 1 \end{array} \right)$$

Ex: Find the augmented coefficient matrix for:

$$\begin{array}{r} x + 2y + 3z = 11 \\ y + 2z = 5 \\ 3z = 6 \end{array} \quad \left( \begin{array}{ccc|c} 1 & 2 & 3 & 11 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 3 & 6 \end{array} \right) \quad \begin{array}{l} 2 \cdot 6 \\ x + 2y + 3z = 11, x = 3 \\ y + 2z = 5, y = 1 \\ 3z = 6, z = 2 \end{array}$$

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Panel 4

Ex: If the augmented matrix of a system of equations is:

$$\left( \begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 5 \end{array} \right) \quad \begin{array}{l} x = 4 \\ y = 2 \\ z = 5 \end{array}$$

what is the solution for the original system?

Ex: Solve a system of equations with augmented matrix

$$\left( \begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 3 & 7 \\ 0 & 0 & 1 & 2 \end{array} \right) \Rightarrow \begin{array}{l} x + z = 2 \\ y + 3z = 7 \\ \underline{z = 2} \end{array} \quad \begin{array}{l} x + 2 = 2 \Rightarrow x = \underline{0} \\ y + 6 = 7 \Rightarrow y = \underline{1} \end{array}$$

nice

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Panel 5

① Eliminate the variable circled from the system:

$$\begin{array}{l}
 2x + y - z = 3 \\
 -2x + 2y + 3z = 8 \\
 x + y + z = 1
 \end{array}
 \left. \begin{array}{l} \\ \\ \end{array} \right\} \Rightarrow
 \begin{array}{l}
 2x + y - z = 3 \\
 3y + 2z = 11 \\
 \textcircled{x} + y + z = 1
 \end{array}$$

next!

$$\begin{pmatrix} 2 & 1 & -1 & 3 \\ -2 & 2 & 3 & 8 \\ 1 & 1 & 1 & 1 \end{pmatrix} \xrightarrow{+} \begin{pmatrix} 2 & 1 & -1 & 3 \\ 0 & 3 & 2 & 11 \\ \textcircled{1} & 1 & 1 & 1 \end{pmatrix} \xrightarrow{f2} \begin{pmatrix} 2 & 1 & -1 & 3 \\ 0 & 3 & 2 & 11 \\ -2 & -2 & -2 & -2 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 & -1 & 3 \\ 0 & 3 & 2 & 11 \\ 0 & -1 & -3 & 5 \end{pmatrix} \xrightarrow{5} \begin{pmatrix} 2 & 1 & -1 & 3 \\ 0 & 3 & 2 & 11 \\ 0 & -3 & -9 & 3 \end{pmatrix} \rightarrow \begin{pmatrix} 2 & 1 & -1 & 3 \\ 0 & 3 & 2 & 11 \\ 0 & 0 & -7 & 14 \end{pmatrix}, z = -2$$

Panel 6

### Elementary Matrix Operations

You are allowed to perform the following operations on an augmented matrix without changing the solution:

- (1) Swap any two rows
- (2) Mult. any row by any number ( $\neq 0$ )
- (3) Replace any row by sum/difference of any two rows

Ex:  $2x + 4y = 8$   
 $-x + y = 2$

$$\left| \begin{pmatrix} 2 & 4 & 8 \\ -1 & 1 & 2 \end{pmatrix} \right. \xrightarrow{+} \left. \begin{pmatrix} 2 & 4 & 8 \\ -2 & 2 & 4 \end{pmatrix} \right. \xrightarrow{+} \left. \begin{pmatrix} 2 & 4 & 8 \\ 0 & 6 & 12 \end{pmatrix} \right. \begin{array}{l} x=0 \\ y=2 \end{array}$$

Panel 7

## Solving a System by Elementary Operations

Solve  $2x + y = 3$

$$x + y + 3z = 1$$

$$x + 2y + 4z = 6$$

$$\left( \begin{array}{cccc} 0 & 1 & 2 & 3 \\ 1 & 1 & 3 & 1 \\ 1 & 2 & 4 & 6 \end{array} \right)$$

*(Note: In the original image, the first column is circled in green, and the first row is crossed out with a red line. The second and third rows are also circled in green.)*

$$x=0, y=7, z=-2$$

① Setup augmented Matrix

② Transform that matrix to  
*upper triangular form*

using elementary matrix operations

③ Read off answers

(Be organised)

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Panel 8

## Legal operations:

① Multiply any row by any number

② Replace any row by sum/difference of rows

③ Switch any two rows

$$\left( \begin{array}{cccc} 0 & 1 & 2 & 3 \\ 1 & 1 & 3 & 1 \\ 1 & 2 & 4 & 6 \end{array} \right) \rightarrow \left( \begin{array}{cccc} 1 & 1 & 3 & 1 \\ 0 & 1 & 2 & 3 \\ -1 & -2 & -4 & -6 \end{array} \right) \rightarrow \left( \begin{array}{cccc} 1 & 1 & 3 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & -1 & -1 & -5 \end{array} \right) \begin{array}{l} \\ \\ 2+ \end{array}$$

$$\rightarrow \left( \begin{array}{cccc|l} 1 & 1 & 3 & 1 & x+y+3z=1, \underline{x=0} \\ 0 & 1 & 2 & 3 & y+2z=3, y=7 \\ 0 & 0 & 1 & -2 & \underline{z=-2} \end{array} \right) \begin{array}{l} (11-6) \\ \\ \end{array}$$

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Panel 9

Solve  $2x + y + z = 3$

$$-x + 2y + 2z = 1$$

$$x - y - 3z = -6$$

$$\begin{pmatrix} 2 & 1 & 1 & 3 \\ -1 & 2 & 2 & 1 \\ 1 & -1 & -3 & -6 \end{pmatrix} \xrightarrow{+} \begin{pmatrix} 2 & 1 & 1 & 3 \\ 0 & 1 & -1 & -5 \\ 0 & -2 & -6 & -9 \end{pmatrix} \xrightarrow{+} \begin{pmatrix} 2 & 1 & 1 & 3 \\ 0 & 1 & -1 & -5 \\ 0 & 0 & -4 & -18 \end{pmatrix} \xrightarrow{+} \begin{pmatrix} 2 & 1 & 1 & 3 \\ 0 & 1 & -1 & -5 \\ 0 & 0 & 7 & 15 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 2 & 1 & 1 & 3 \\ 0 & -3 & 3 & 15 \\ 0 & 3 & 7 & 15 \end{pmatrix} \xrightarrow{\frac{1}{3}} \begin{pmatrix} 2 & 1 & 1 & 3 \\ 0 & -1 & 1 & 5 \\ 0 & 0 & 10 & 30 \end{pmatrix} \xrightarrow{\frac{1}{10}} \begin{pmatrix} 2 & 1 & 1 & 3 \\ 0 & -1 & 1 & 5 \\ 0 & 0 & 1 & 3 \end{pmatrix} \begin{array}{l} x=1 \\ y=-2 \\ z=3 \end{array}$$

Gauss-Jordan-Elimination Method.

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Panel 10

Ex: Solve the following system of linear equations:

$$x + y + z = 0$$

$$-x + 2y + z = 3$$

$$-x - 4y - 2z = 0$$

HW

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Panel 11

$$\begin{aligned}\underline{\text{Ex:}} \quad & 2x + y + 6z = 3 \\ & x - y + 4z = 1 \\ & 3x + 2y - 2z = 2\end{aligned}$$

HW

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Panel 12

$$\begin{aligned}\underline{\text{Ex:}} \quad & 5p + 11q = 7 \\ & 10p + 22q = 33\end{aligned}$$

HW

$$\begin{aligned}\underline{\text{Ex:}} \quad & x - y + 2z = 0 \\ & 2x + y - z = 0 \\ & x + 2y - 3z = 0\end{aligned}$$

Quizzes on Monday

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Panel 13

$$\lim_{x \rightarrow 0} \frac{\sin(2x)}{x}$$

$$f(x) = x(x-4)^2(x+4)^3, \text{ find } f'(1.7)$$