Panel 1

Last Topic: Solving Systems of Equations

Want to solve $3x - y = 1$
$x + 2y = 5$

Substitution

Solve equation 1 for y, subst. into equation 2

Elimination

Mult. (1) by 2 and add to (2) to make the y drop out!

1

Panel 2

Matrix Method

$3x - y = 1$
$x + 2y = 5$

$$\begin{matrix} x & y \end{matrix}$$
$$\begin{pmatrix} 3 & -1 \\ 1 & 2 \end{pmatrix}$$

2×2 matrix (↑ rows, ↑ columns)

Definition: (Coefficient Matrix)
the coefficients in front of variables of a linear system of equations, arranged in rows and columns, akin a table

Def: (Augmented coefficient matrix)
the coefficient matrix with an extra column for the constant terms.

$$\begin{matrix} x & y & = \# \end{matrix}$$
$$\left(\begin{array}{cc|c} 3 & -1 & 1 \\ 1 & 2 & 5 \end{array}\right)$$

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Panel 3

Ex: Find the augmented coefficient matrix for the system:

$$\begin{aligned} 2x + 1y - z &= 3 \\ -2x + 2y + 3z &= 8 \\ x + y + z &= 1 \end{aligned} \qquad \left(\begin{array}{cccc} 2 & 1 & -1 & 3 \\ -2 & 2 & 3 & 8 \\ 1 & 1 & 1 & 1 \end{array}\right) \quad 3\times 4 \text{ matrix}$$

Ex: Find the augmented coefficient matrix for:

$$\begin{aligned} x + 2y + 3z &= 11 \\ y + 2z &= 5 \\ 3z &= 6 \end{aligned} \qquad \left(\begin{array}{cccc} 1 & 2 & 3 & 11 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 3 & 6 \end{array}\right)$$

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Panel 4

Ex: If the augmented matrix of a system of equations is:

$$\left(\begin{array}{cccc} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 5 \end{array}\right) \qquad \begin{aligned} x &= 4 \\ y &= 2 \\ z &= 5 \end{aligned}$$

what is the solution for the original system?

Ex: Solve a system of equations with augmented matrix

$$\left(\begin{array}{cccc} 1 & 0 & 1 & 2 \\ 0 & 1 & 3 & 7 \\ 0 & 0 & 1 & 2 \end{array}\right) \qquad \begin{aligned} x + z &= 2 \\ y + 3z &= 7 \\ z &= 2 \end{aligned}$$

$x + 2 = 2 \Rightarrow x = 0$

$y + 6 = 7 \Rightarrow y = 1$

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Panel 5

So: If an augmented matrix is:

(1) $\begin{pmatrix} 2 & 1 & -1 & 3 \\ -2 & 2 & 3 & 8 \\ 1 & 1 & 1 & 1 \end{pmatrix}$

Is hard to solve. But

$x \quad y \quad z \quad =$

(2) $\begin{pmatrix} 1 & 2 & 3 & 11 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 3 & 6 \end{pmatrix}$ $\Rightarrow$ $x = 3$, $y = 1$, $z = 2$

How to convert (1) into (2) without changing the solution?

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Panel 6

① Eliminate the variable circled from the system:

$$\begin{matrix} 2x + y - z = 3 \\ -2x + 2y + 3z = 8 \\ x + y + z = 1 \end{matrix} \Rightarrow \begin{matrix} 2x + y - z = 3 \\ 3y + 2z = 11 \\ x + y + z = 1 \end{matrix}$$

$$\begin{pmatrix} 2 & 1 & -1 & 3 \\ -2 & 2 & 3 & 8 \\ 1 & 1 & 1 & 1 \end{pmatrix} \overset{(1)+(2)\to(2)}{\Rightarrow} \begin{pmatrix} 2 & 1 & -1 & 3 \\ 0 & 3 & 2 & 11 \\ -2 & -2 & -2 & -2 \end{pmatrix} \Rightarrow \begin{pmatrix} 2 & 1 & -1 & 3 \\ 0 & 3 & 2 & 11 \\ 0 & -1 & -3 & 1 \end{pmatrix}$$

still to do

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Panel 7

Elementary Matrix Operations

You are allowed to perform the following operations on an augmented matrix without changing the solution:

(1) Swap any two rows
(2) Multiply any row by any non-zero number
(3) Replace any row by sum/difference of two rows.

Ex: $2x+4y=8$, $-x+y=2$

$$\begin{pmatrix} 2 & 4 & 8 \\ -1 & 1 & 2 \end{pmatrix} \xrightarrow{2\cdot} \begin{pmatrix} 2 & 4 & 8 \\ -2 & 2 & 4 \end{pmatrix} \xrightarrow{(1)+(2)} \begin{pmatrix} 2 & 4 & 8 \\ 0 & 6 & 12 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 4 \\ 0 & 1 & 2 \end{pmatrix} \Rightarrow \begin{matrix} x=0 \\ y=2 \end{matrix}$$

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Panel 8

Solving a System by Elementary Operations

Solve $2z+y=3$, $x+y+3z=1$, $x+2y+4z=6$

$$\begin{pmatrix} 0 & 1 & 2 & 3 \\ 1 & 1 & 3 & 1 \\ 1 & 2 & 4 & 6 \end{pmatrix}$$

(1) Setup augmented matrix
(2) Turn this matrix into upper triangular matrix by elementary matrix ops
(3) Read off answer

$x=0$
$y=7$, $z=-2$

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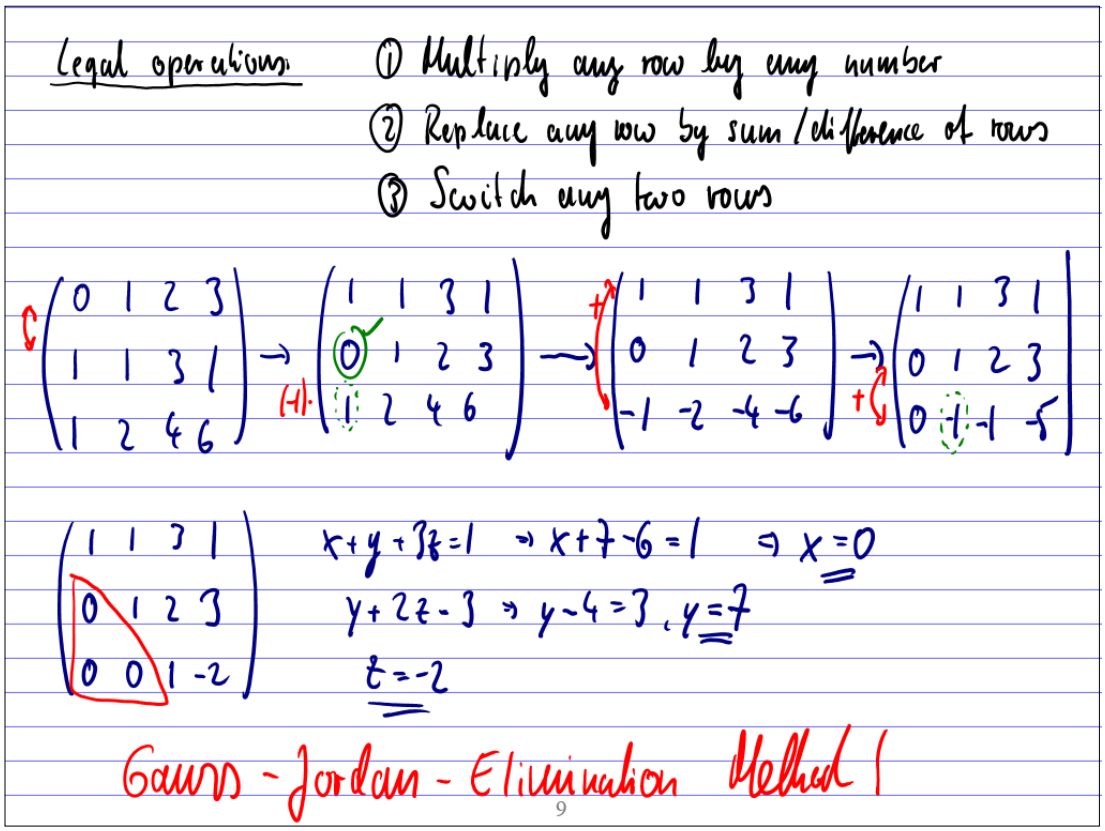
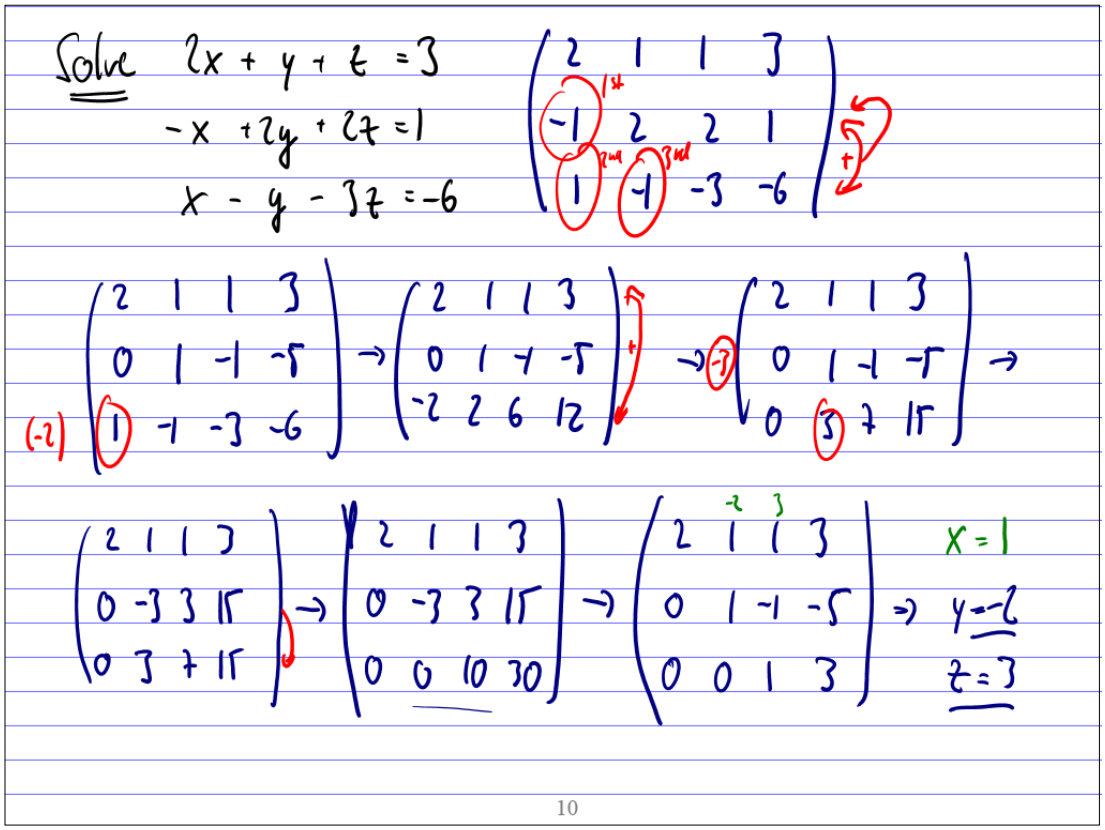
Panel 9



Panel 10



Panel 11

Ex: Solve the following system of linear equations:

$$x + y + z = 0$$
$$-x + 2y + z = 3$$
$$-x - 4y - 2z = 0$$

HW

11

Panel 12

Ex:

$$2x + y + 6z = 3$$
$$x - y + 4z = 1$$
$$3x + 2y - 2z = 2$$

HW

12

Panel 13

Ex: $5p + 11q = 7$

$10p + 22q = 33$

Ex: $x - y + 2z = 0$

$2x + y - z = 0$

$x + 2y - 3z = 0$

HW

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Panel 14

$\lim_{x \to 0} \frac{\sin(2x)}{x}$

$f(x) = x(x-2)^4(x+2)^3$

$f'(1.3)$

$f''(1.7)$

critical points

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