

Panel 1

Last Topic: Solving Systems of Equations

Want to solve

$$\begin{aligned} 3x - y &= 1 \\ x + 2y &= 5 \end{aligned}$$

Substitution

Solve equation 1 for y ,
subst. into equation 2

Elimination

Mult. (1) by 2 and
add to (2) to make the
 y drop out!

1

Panel 2

Matrix Method

$$\begin{aligned} 3x - y &= 1 \\ x + 2y &= 5 \end{aligned}$$

$$\begin{matrix} x & y \\ \left(\begin{array}{cc} 3 & -1 \\ 1 & 2 \end{array} \right) \end{matrix}$$

↖ 2x2 matrix
↑ row
↑ column

Definition: (Coefficient Matrix)

The coefficients in front of variable terms of a linear system of equations, arranged in rows and columns, aka. a table

Def: (Augmented coefficient matrix)

the coefficient matrix with an extra column
for the constant terms.

$$\begin{matrix} x + y = \# \\ \left(\begin{array}{cc|c} 3 & -1 & 1 \\ 1 & 2 & 5 \end{array} \right) \end{matrix}$$

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Panel 3

Ex: Find the augmented coefficient matrix for the system:

$$\begin{array}{r} 2x + y - z = 3 \\ -2x + 2y + 3z = 8 \\ x + y + z = 1 \end{array} \quad \left(\begin{array}{cccc} 2 & 1 & -1 & 3 \\ -2 & 2 & 3 & 8 \\ 1 & 1 & 1 & 1 \end{array} \right) \quad 3 \times 4 \text{ matrix}$$

Ex: Find the augmented coefficient matrix for:

$$\begin{array}{r} x + 2y + 3z = 11 \\ y + 2z = 5 \\ 3z = 6 \end{array} \quad \left(\begin{array}{cccc} 1 & 2 & 3 & 11 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 3 & 6 \end{array} \right)$$

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Panel 4

Ex: If the augmented matrix of a system of equations is:

$$\left(\begin{array}{cccc} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 5 \end{array} \right) \quad \begin{array}{l} x = 4 \\ y = 2 \\ z = 5 \end{array}$$

what is the solution for the original system?

Ex: Solve a system of equations with augmented matrix

$$\left(\begin{array}{cccc} 1 & 0 & 1 & 2 \\ 0 & 1 & 3 & 7 \\ 0 & 0 & 1 & 2 \end{array} \right) \quad \begin{array}{l} x + z = 2 \quad \leftarrow x + z = 2, \underline{x=0} \\ y + 3z = 7 \quad \leftarrow y + 6 = 7 \Rightarrow \underline{y=1} \\ \underline{z=2} \end{array}$$

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Panel 5

So: If an augmented matrix is:

$$(1) \begin{pmatrix} 2 & 1 & -1 & 3 \\ -2 & 2 & 3 & 8 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

Is hard to solve. But

$$(2) \begin{pmatrix} 1 & 2 & 3 & 11 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 3 & 6 \end{pmatrix} \Rightarrow \begin{array}{l} \underline{x=3} \\ \underline{y=1} \\ \underline{z=2} \end{array}$$

How to convert (1) into (2) without changing the solution?

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Panel 6

① Eliminate the variable circled from the system:

$$\left. \begin{array}{l} 2x + y - z = 3 \\ -2x + 2y + 3z = 8 \\ \textcircled{x} + y + z = 1 \end{array} \right\} \Rightarrow \begin{array}{l} 2x + y - z = 3 \\ 3y + 2z = 11 \\ x + y + z = 1 \end{array}$$

$$\begin{pmatrix} 2 & 1 & -1 & 3 \\ -2 & 2 & 3 & 8 \\ 1 & 1 & 1 & 1 \end{pmatrix} \xrightarrow{(1)+(2) \rightarrow (2)} \begin{pmatrix} 2 & 1 & -1 & 3 \\ 0 & 3 & 2 & 11 \\ -2 & -2 & -2 & -2 \end{pmatrix} \Rightarrow \begin{pmatrix} 2 & 1 & -1 & 3 \\ 0 & 3 & 2 & 11 \\ 0 & \textcircled{-1} & -3 & 1 \end{pmatrix}$$

still to do

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Panel 7

Elementary Matrix Operations

You are allowed to perform the following operations on an augmented matrix without changing the solution:

- (1) Swap any two rows
- (2) Multiply any row by any non-zero number
- (3) Replace any row by sum/difference of two rows.

Ex: $2x + 4y = 8$
 $-x + y = 2$

$$\left(\begin{array}{cc|c} 2 & 4 & 8 \\ -1 & 1 & 2 \end{array} \right) \rightarrow \left(\begin{array}{cc|c} 2 & 4 & 8 \\ -2 & 2 & 4 \end{array} \right) \xrightarrow{(1)+(2)} \left(\begin{array}{cc|c} 2 & 4 & 8 \\ 0 & 6 & 12 \end{array} \right)$$

$$\rightarrow \left(\begin{array}{cc|c} 1 & 2 & 4 \\ 0 & 1 & 2 \end{array} \right) \Rightarrow \begin{array}{l} x=0 \\ y=2 \end{array}$$

Panel 8

Solving a System by Elementary Operations

Solve $2z + y = 3$
 $x + y + 3z = 1$
 $x + 2y + 4z = 6$

$$\left(\begin{array}{ccc|c} 0 & 1 & 2 & 3 \\ 1 & 1 & 3 & 1 \\ 1 & 2 & 4 & 6 \end{array} \right)$$

(Note: In the original image, the first column is circled in blue, and the first two rows are circled in red. The first row is labeled '1st' and the second row is labeled '2nd'. The second row is also labeled '3rd' in red.)

- ① Setup augmented matrix
- ② Turn this matrix into upper triangular matrix by elementary matrix ops
- ③ Read off answer

$$\begin{array}{l} x=0 \\ y=7, z=-2 \end{array}$$

Panel 9

Legal operations:

- ① Multiply any row by any number
- ② Replace any row by sum/difference of rows
- ③ Switch any two rows

$$\begin{pmatrix} 0 & 1 & 2 & 3 \\ 1 & 1 & 3 & 1 \\ 1 & 2 & 4 & 6 \end{pmatrix} \xrightarrow{(-1) \cdot R_1} \begin{pmatrix} 1 & 1 & 3 & 1 \\ 0 & 1 & 2 & 3 \\ 1 & 2 & 4 & 6 \end{pmatrix} \xrightarrow{(-1) \cdot R_2} \begin{pmatrix} 1 & 1 & 3 & 1 \\ 0 & 1 & 2 & 3 \\ -1 & -2 & -4 & -6 \end{pmatrix} \xrightarrow{+R_2} \begin{pmatrix} 1 & 1 & 3 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & -1 & -5 & -5 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 3 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & -2 \end{pmatrix} \quad \begin{aligned} x + y + 3z &= 1 \Rightarrow x + 7 - 6 = 1 \Rightarrow \underline{x = 0} \\ y + 2z &= 3 \Rightarrow y - 4 = 3, \underline{y = 7} \\ z &= -2 \end{aligned}$$

Gauss-Jordan-Elimination Method I

Panel 10

Solve $2x + y + z = 3$

$-x + 2y + 2z = 1$

$x - y - 3z = -6$

$$\begin{pmatrix} 2 & 1 & 1 & 3 \\ -1 & 2 & 2 & 1 \\ 1 & -1 & -3 & -6 \end{pmatrix} \xrightarrow{+R_1} \begin{pmatrix} 2 & 1 & 1 & 3 \\ 0 & 3 & 3 & 7 \\ 0 & -2 & -4 & -9 \end{pmatrix} \xrightarrow{+R_2} \begin{pmatrix} 2 & 1 & 1 & 3 \\ 0 & 3 & 3 & 7 \\ 0 & 1 & -1 & -5 \end{pmatrix} \xrightarrow{+R_2} \begin{pmatrix} 2 & 1 & 1 & 3 \\ 0 & 1 & -1 & -5 \\ 0 & 2 & 2 & 1 \end{pmatrix} \xrightarrow{+R_2} \begin{pmatrix} 2 & 1 & 1 & 3 \\ 0 & 1 & -1 & -5 \\ 0 & 0 & 0 & -9 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 & 1 & 3 \\ 0 & 1 & -1 & -5 \\ 0 & 0 & 0 & -9 \end{pmatrix} \xrightarrow{(-2) \cdot R_2} \begin{pmatrix} 2 & 1 & 1 & 3 \\ 0 & 1 & -1 & -5 \\ 0 & 0 & 0 & -9 \end{pmatrix} \xrightarrow{+R_2} \begin{pmatrix} 2 & 1 & 1 & 3 \\ 0 & 1 & -1 & -5 \\ 0 & 0 & 0 & -9 \end{pmatrix} \xrightarrow{+R_2} \begin{pmatrix} 2 & 1 & 1 & 3 \\ 0 & 1 & -1 & -5 \\ 0 & 0 & 0 & -9 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 & 1 & 3 \\ 0 & 1 & -1 & -5 \\ 0 & 0 & 0 & -9 \end{pmatrix} \xrightarrow{+R_2} \begin{pmatrix} 2 & 1 & 1 & 3 \\ 0 & 1 & -1 & -5 \\ 0 & 0 & 0 & -9 \end{pmatrix} \xrightarrow{+R_2} \begin{pmatrix} 2 & 1 & 1 & 3 \\ 0 & 1 & -1 & -5 \\ 0 & 0 & 0 & -9 \end{pmatrix} \Rightarrow \begin{aligned} x &= 1 \\ y &= -2 \\ z &= 3 \end{aligned}$$

Panel 11

Ex: Solve the following system of linear equations:

$$x + y + z = 0$$

$$-x + 2y + z = 3$$

$$-x - 4y - 2z = 0$$

HW

11

Panel 12

Ex: $2x + y + 6z = 3$

$$x - y + 4z = 1$$

$$3x + 2y - 2z = 2$$

HW

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Panel 13

$$\underline{\text{Ex:}} \quad 5p + 11q = 7$$

$$10p + 22q = 33$$

$$\underline{\text{Ex:}} \quad x - y + 2z = 0$$

$$2x + y - z = 0$$

$$x + 2y - 3z = 0$$

HLW

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Panel 14

$$\lim_{x \rightarrow 0} \frac{\sin(2x)}{x}$$

$$f(x) = x(x-2)^4(x+2)^3$$

$$f'(1.7)$$

$$f''(1.7)$$

critical points

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