

Panel 1

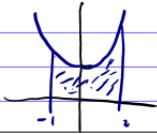
Last Time.

Area under curve: $\int_a^b f(x) dx$ if $f(x) > 0$

Area between curves: $\int_a^b f(x) - g(x) dx$ if $f(x) > g(x)$

Ex: Area under $f(x) = x^2 + 1$ from $x = -1$ to $x = 2$

$$\int_{-1}^2 x^2 + 1 dx = \left. \frac{1}{3}x^3 + x \right|_{-1}^2 = \left[\frac{1}{3}(2^3 + 2) \right] - \left[\frac{1}{3}(-1^3 - 1) \right] = \frac{8}{3} + 2 + \frac{1}{3} + 1 = \underline{\underline{6}}$$

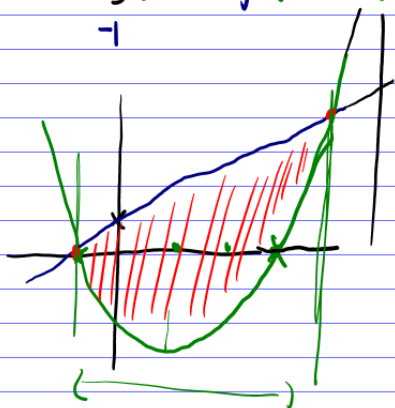


1

Panel 2

Area between $f(x) = x^2 - (x-3)(x+1)$ and $g(x) = x+1$ from -1 to 5

$$A = \int_{-1}^4 (x+1) - (x^2 - 2x - 3) dx = 4$$



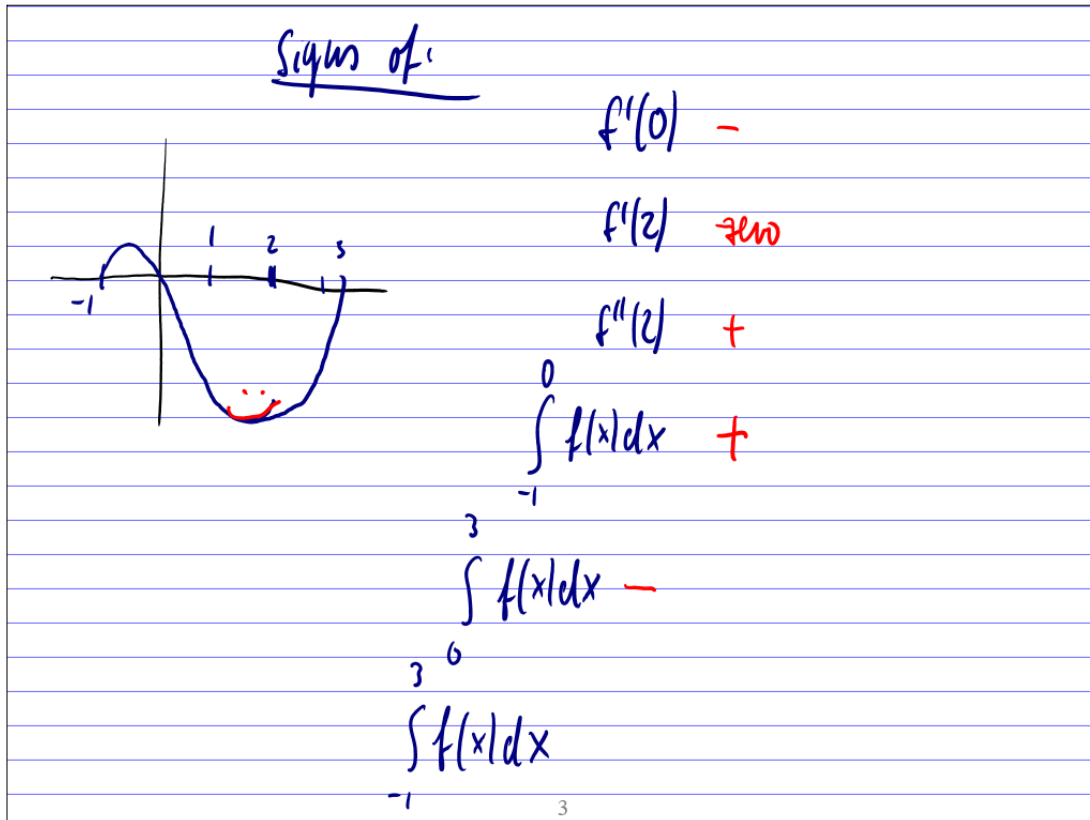
$$x^2 - 2x - 3 = x + 1$$

$$x^2 - 3x - 4 = 0$$

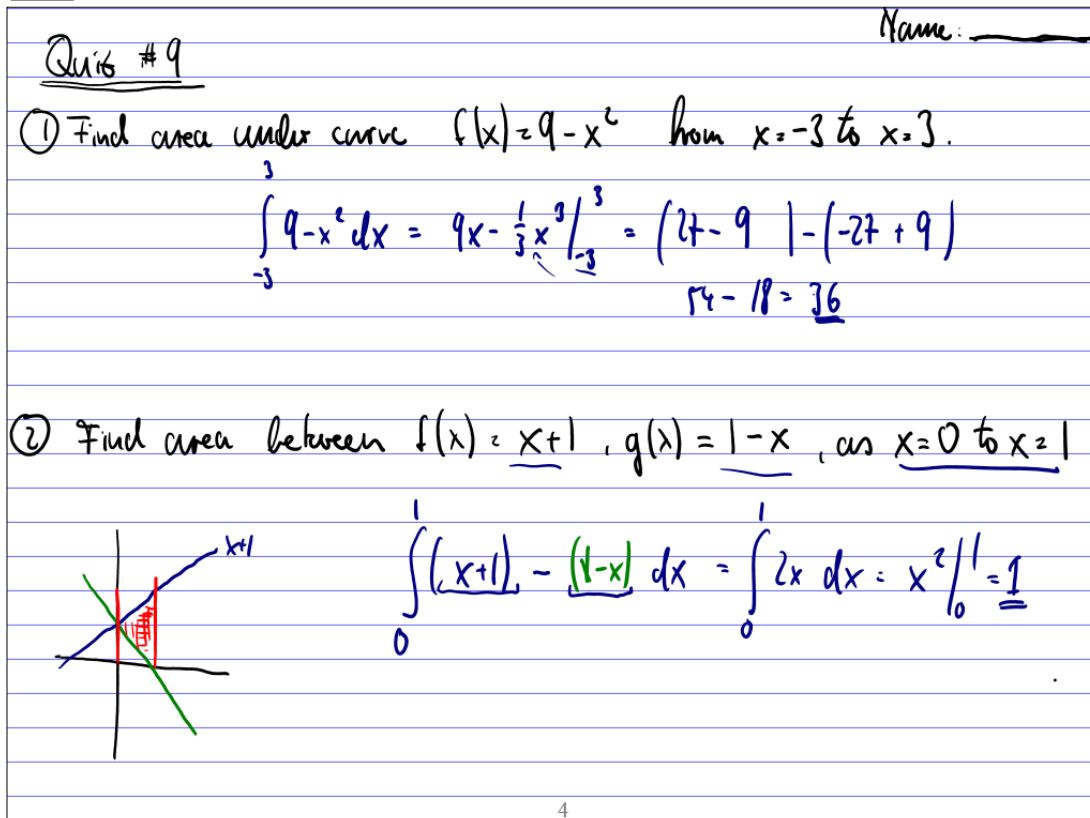
$$(x-4)/(x+1) = 0 \Rightarrow x = \underline{\underline{-1, 4}}$$

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Panel 3



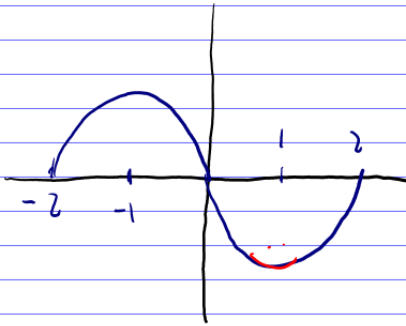
Panel 4



Panel 5

③ Consider the function $f(x)$ whose graph is shown below.

Find the signs (pos, neg, zero) of:



a) $f'(0)$ -

b) $f''(1)$ +

c) $\int_{-2}^0 f(x) dx$ +

d) $\int_{-2}^2 f(x) dx$ zero

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Panel 6

Quiz #9

Name: _____

① Find area under curve $f(x) = 9 - x^2$ from $x = -3$ to $x = 3$.

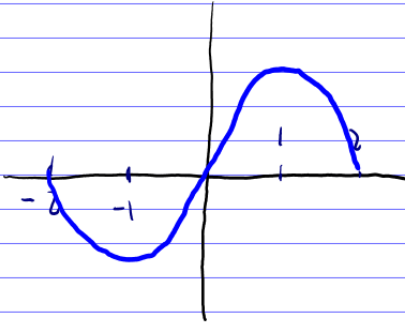
② Find area between $f(x) = x + 1$, $g(x) = 1 - x$, as $x = 0$ to $x = 1$

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Panel 7

③ Consider the function $f(x)$ whose graph is shown below.

Find the signs (pos, neg, zero) of:



a) $f'(0)$

b) $f''(1)$

c) $\int_{-2}^0 f(x) dx$

d) $\int_{-2}^2 f(x) dx$

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Panel 8

Final Topics: Financial Mathematics

Recall Compound Interest Formula: If you invest a principal P at an interest rate r per period compounded for n periods in total, you have:

$$S = P(1+r)^n$$

Ex: \$2000 at 8% for 6 years compounded quarterly.

$$S = 2000 \left(1 + \frac{0.08}{4}\right)^{24} = \underline{\hspace{2cm}}$$

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Panel 9

If you invest \$P at 10% compounded quarterly for one year, it will earn more than 10% that year.

$$S - P = P \left(1 + \frac{0.1}{4}\right)^4 - P = P \left(\left(1 + \frac{0.1}{4}\right)^4 - 1 \right) = P (1.025^4 - 1) =$$

$$= P \cdot 0.1039 \text{ or}$$

$$\text{10.39\% of } P.$$

Def: The effective rate r_e equivalent to a nominal rate r compounded n times per year is:

$$r_e = \left(1 + \frac{r}{n}\right)^n - 1$$

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Panel 10

$$\text{Effective rate } r_e = \left(1 + \frac{r}{n}\right)^n - 1$$

Ex: Find effective rate equivalent to 6% compounded
(a) semiannually, (b) monthly

$$a) \left(1 + \frac{0.06}{2}\right)^2 - 1 = 0.0609 \text{ or } \underline{6.09\%}$$

$$b) \left(1 + \frac{0.06}{12}\right)^{12} - 1 = 0.06167 \text{ or } \underline{\underline{6.17\%}}$$

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Panel 11

Bank A offers 10% compounded weekly
 B offers 10.3% compounded semiannually.

$$\underline{A}: r_e = \left(1 + \frac{0.1}{52}\right)^{52} - 1 = 0.105 \text{ or } \underline{10.5\%}$$

$$r_e = \left(1 + \frac{0.103}{2}\right)^2 - 1 = 0.1056 \text{ or } \underline{10.56\%}$$

Choose Bank B HW! \$2000 either bank, 10 years

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Panel 12

Present Value:

Usually invest P\$ for X years and see what you get.

Want to have a fixed amount of money after X years.

How much to invest today!

→ Present Value:

12

Panel 13

Present Value

X

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Panel 14

Ex: Suppose you need \$1,000 in three years, and your bank offers 9% compounded monthly. Find the present value of \$1,000 in 3 years.

$$S = P(1 + r)^n$$

$$1000 = P \left(1 + \frac{0.09}{12} \right)^{36}$$

$$\frac{1000}{\left(1 + \frac{0.09}{12} \right)^{36}} = \underline{\underline{\$764.15}}$$

Present value of \$1000 is
\$764.15 at 9% comp. monthly!

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Panel 15

Present Value	<i>The PV function returns the present value of an investment. The present value is the total amount that a series of future payments is worth now. For example, when you borrow money, the loan amount is the present value to the lender.</i>	
<i>Excel Formula:</i>	=PV(rate, nper, pmt, fv), where	
	rate:	is the interest rate per period.
	nper:	is the total number of payment periods in an annuity.
	pmt:	is the payment made each period; it cannot change over the life of the annuity.
	fv:	is the future value, or a cash balance you want to attain after the last payment is made.