Panel 1

Last Time

Indefinite Integral

$\int f(x)\,dx = F(x) + C$, where F is antiderivative, i.e. $F'(x) = f(x)$

bounds

Definite Integral

$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$

represents graphically area under curve, as long as $f(x) \geq 0$

1

Panel 2

Ex: $\int x^2 + \sqrt{x^3} + 1/x \, dx = \frac{1}{3}x^3 + \frac{2}{5}x^{5/2} + \ln|x| + C$ ($\sqrt{x^3} = x^{3/2}$)

$\int_0^1 4x - 9x^2\,dx = 2x^2 - 3x^3\Big|_0^1 = (2-3) - (0-0) = -1$

Ex: Area under $f(x) = 3x^2 + 1$ from $x = 1$ to 2

$\int_1^2 3x^2 + 1\,dx = x^3 + x\Big|_1^2 = (8+2) - (1+1) = 8$

positive

2

Panel 3

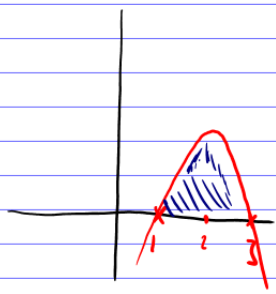
Ex: Area under curve $f(x) = -x^2 + 4x - 3 = -(x^2 - 4x + 3) = -(x-1)(x-3)$

$\frac{-4}{-2} = 2 = x$

$$\int_1^3 -x^2 + 4x - 3\,dx$$

$$= -\frac{1}{3}x^3 + 2x^2 - 3x \Big|_1^3 = (-9 + 18 - 9) - (-\tfrac{1}{3} + 2 - 3) =$$

$$= 0 + \tfrac{1}{3} - 2 + 3 = \tfrac{4}{3}$$

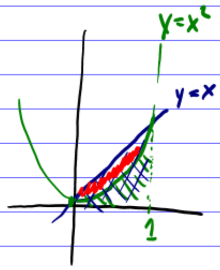


3

Panel 4

Area between Curves

Find area between $y = x$ and $y = x^2$ from $x = 0$ to $x = 1$



$x^2 = x \Rightarrow x^2 - x = 0 \Rightarrow x(x-1) = 0 \Rightarrow x = 0, 1$

green area $\int_0^1 x^2\,dx$

blue area $\int_0^1 x\,dx$

Total area (red) = blue − green $= \int_0^1 x - x^2\,dx = \frac{1}{2}x^2 - \frac{1}{3}x^3 \Big|_0^1 = \frac{1}{2} - \frac{1}{3} - 0 = \frac{1}{6}$

Thm: Area between $f(x)$ and $g(x)$ from a to b is $\int_a^b f(x) - g(x)\,dx$

as long as $f(x) \geq g(x)$

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Panel 5

Ex: Area between $f(x) = x^2$ and $g(x) = x$ from $x=0$ to $x=1$

done already

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Panel 6

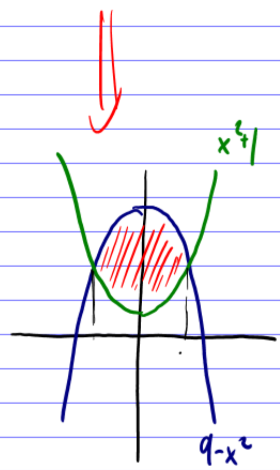
Ex: Area between $9-x^2$ and x^2+1

① $\int_{-2}^{2} (9-x^2)-(x^2+1)\,dx = \int_{-2}^{2} 8-2x^2\,dx = 8x-\frac{2}{3}x^3\Big|_{-2}^{2} =$

② $\int (x^2+1) - (9-x^2)\,dx$

$\left(16-\frac{16}{3}\right)-\left(-16+\frac{16}{3}\right) =$

$= 32-\frac{32}{3} = \frac{64}{3}$



$9-x^2 = x^2+1$

$8-2x^2=0$

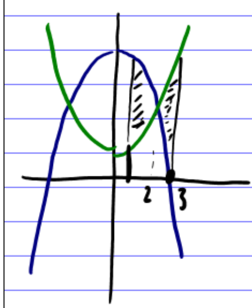
$2(4-x^2)=0, \ x=\pm 2$

6

Panel 7

Ex: Area between $y = 9 - x^2$ and $y = x^2 + 1$ from $x = 1$ to $x = 3$

$$\int_1^3 (9-x^2) - (x^2+1)\,dx$$



$$\int_1^2 (9-x^2)-(x^2+1)\,dx + \int_2^3 (x^2+1)-(9-x^2)\,dx =$$

$$= \int_1^2 8 - 2x^2\,dx + \int_2^3 2x^2 - 8\,dx =$$

$$= 8x - \frac{2}{3}x^3 \Big|_1^2 + \frac{2}{3}x^3 - 8x \Big|_2^3$$

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Panel 8

Ex: Suppose the marginal cost for producing q items is $\frac{dC}{dq} = 0.6q + 2$. Production is set to $q = 80$ units. What is the cost to raise production to $q = 100$ units?

$$C(100) - C(80) = C(x)\Big|_{80}^{100} = \int_{80}^{100} C'(x)\,dx =$$

$$\Rightarrow \quad = \int_{80}^{100} 0.6q + 2\,dq =$$

$$= 0.3q^2 + 2q \Big|_{80}^{100} = (0.3 \cdot 100^2 + 2 \cdot 100) - (0.3 \cdot 80^2 + 2 \cdot 80)$$

calculator

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Panel 9

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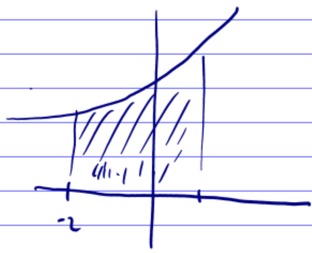
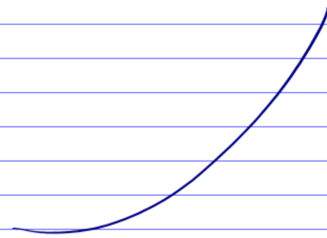
6. Find the area under the graph $y = e^x + 2$ from x = -2 to x = 1. Sketch and shade the region. (6%)

$$\int_{-2}^{1} e^x + 2\, dx = e^x + 2x\Big|_{-2}^{1} = (e+2) - (e^{-2} - 4) = e + 6 - e^{-2}$$





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Panel 10

9. Suppose the marginal cost of making q throw rugs is $c' = 8q - 3\sqrt{q} + 4e^q$, and the fixed cost is $4400. Find the formula for the cost function. (6%)

$$C(q) = \int 8q - 3\sqrt{q} + 4e^q\, dq =$$ ($\sqrt{q} = q^{1/2}$)

$$4q^2 - 3\tfrac{2}{3}q^{3/2} + 4e^q + C =$$

$$= 4q^2 - 2q^{3/2} + 4e^q + C$$

$$C(0) = 0 - 0 + 4 + C = 4400$$

$$C = 4396$$

$$C(q) = 4q^2 - 2q^{3/2} + 4e^q + 4396$$

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Panel 11

Final Topics: Financial Mathematics

Recall Compound Interest Formula: If you invest a principal P at an interest rate r per period compounded for n periods in total, you have:

$$S = P(1+r)^n \quad , r \text{ rate per period}$$

Ex: \$1000 at APR 8% compounded quarterly over 5 years

$$S = 1000\left(1+\frac{0.08}{4}\right)^{20} = \$\,1485.95$$

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Panel 12

Ex: Suppose \$500 is compounded semi-annually over 3 years and amounts to \$588.38. What is the nominal interest rate?

$$S = P(1+r)^n$$

$$588.38 = 500(1+r/2)^6$$

$$r = 5.5\%$$

$$\frac{588.38}{500} = \left(1+\frac{r}{2}\right)^6$$

$$1.17676 = (1+r/2)^6 \quad \Big| \sqrt[6]{}$$

$$\sqrt[6]{1.17676} = 1 + r/2 \Rightarrow 1.0275 = 1 + r/2 \Rightarrow 0.0275 = r/2 \Rightarrow r = 0.055$$

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Panel 13

Ex: How long will it take for \$600 to amount to \$900 at APR of 6% compounded quarterly?

$$S = P(1+r)^n$$

$$900 = 600\left(1 + \frac{0.06}{4}\right)^{4t} \quad , t = \# \text{ years}$$

$$1.5 = \frac{900}{600} = (1.015)^{4t}$$

$$1.5 = (1.015)^{4t} \quad | \ln(\ )$$

$$\ln(1.5) = \ln(1.015^{4t})$$

$$\ln(1.5) = 4t \ln(1.015) \Rightarrow 4t = \frac{\ln(1.5)}{\ln(1.015)} = 27.23$$

$$t = \frac{27.23}{4} = 6.8$$

t years

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Panel 14

Quiz on Wed — Integration only, i.e. area

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