Panel 1

Last Time

Indefinite integral

$\int f(x)\,dx = F(x) + C$, where F is antiderivative, i.e. $F'(x) = f(x)$

bounds

definite Integral

$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$

represents graphically area under curve, as long as $f(x) \geq 0$

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Panel 2

Ex: $\int x^2 + \sqrt{x^3} + 1/x \, dx = \frac{1}{3}x^3 + \frac{2}{5}x^{5/2} + \ln(x) + C$ (function)

($\sqrt{x^3} = x^{3/2}$)

$\int_0^1 4x - 9x^2 \, dx = 2x^2 - \frac{3x^3}{2}\Big|_0^1 = (2 - 3) - (0 - 0) = -1$ (number)

Ex: Area under $f(x) = 3x^2 + 1$ from $x = 1$ to 2

$A = \int_1^2 3x^2 + 1 \, dx = x^3 + x\Big|_1^2 = (8 + 2) - (1 + 1) = \underline{\underline{8}}$

area because $3x^2 + 1$ is positive

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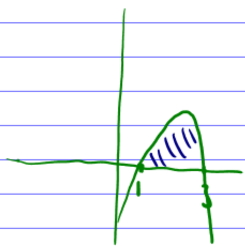
Panel 3

Ex: Area under curve $f(x) = -x^2 + 4x - 3 = -(x-1)(x-3)$

$f'(x) = -(2x-4) = 0 \Rightarrow x = 2$

$$\int_1^3 -x^2 - 4x + 3\,dx = -\frac{1}{3}x^3 - 2x^2 + 3x \Big|_1^3 = (-9 - 18 + 9) - (-\frac{1}{3} - 2 + 3)$$



! Sometimes you need to find the bounds of integration by graphing the function

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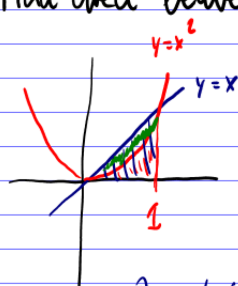
Panel 4

Area between Curves

Find area between $y = x$ and $y = x^2$ from $x = 0$ to $x = 1$



red area : $\int_0^1 x^2\,dx$

blue area : $\int_0^1 x\,dx$

Area between $A = \int_0^1 x - x^2\,dx = \frac{1}{2}x^2 - \frac{1}{3}x^3 \Big|_0^1 = \frac{1}{2} - \frac{1}{3} - 0 = \frac{1}{6}$

Theorem: The area between $f(x)$ and $g(x)$ from a to b is:

$$\int_a^b f(x) - g(x)\,dx \quad , \text{as long as } f(x) \geq g(x)$$

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Panel 5

Ex: Area between $f(x) = x^2$ and $g(x) = x$ from $x = 0$ to $x = 1$

done

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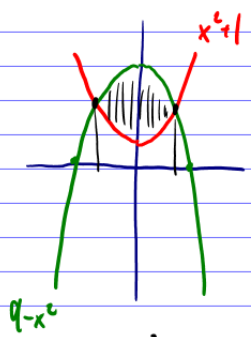
Panel 6

Ex: Area between $9 - x^2$ and $x^2 + 1$

Answer $\int_{?}^{?} (9-x^2) - (x^2+1)\, dx$ ① or $\int \cancel{(x^2+1) - (9-x^2)}\, dx$ ②



to find the points of intersection:

$9 - x^2 = x^2 + 1$

$8 - 2x^2 = 0$

$2(4 - x^2) = 0 \Rightarrow x = \pm 2$

Area: $\int_{-2}^{2} (9-x^2) - (x^2+1)\, dx = \int_{-2}^{2} 8 - 2x^2\, dx = 8x - \frac{2}{3}x^3 \Big|_{-2}^{2} =$

$\left(16 - \frac{16}{3}\right) - \left(-16 + \frac{16}{3}\right) = 32 - \frac{32}{3} = \frac{64}{3}$

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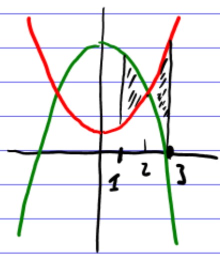
Panel 7

Ex: Area between $y = 9 - x^2$ and $y = x^2 + 1$ from $x = 1$ to $x = 3$



$$\cancel{\int_1^3 (9-x^2) - (x^2+1)\, dx}$$

$$A = \int_1^2 (9-x^2) - (x^2+1)\, dx + \int_2^3 (x^2+1) - (9-x^2)\, dx$$

$$= \int_1^2 8 - 2x^2\, dx + \int_2^3 2x^2 - 8\, dx$$

$$= 8x - \frac{2}{3}x^3 \Big|_1^2 + \frac{2}{3}x^3 - 8x \Big|_2^3 = \text{Mess}$$

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Panel 8

Ex: Suppose the marginal cost for producing q items is $\frac{dC}{dq} = 0.6q + 2$. Production is set to $q = 80$ units. What is the cost to raise production to $q = 100$ units?

Want: $C(100) - C(80) = C(x)\Big|_{80}^{100} = \int_{80}^{100} C'(x)\, dx$

$$= \int_{80}^{100} 0.6q + 2\, dq =$$

Appli

$$0.3q^2 + 2q\Big|_{80}^{100} = (0.3(100)^2 + 2\cdot 100) - (0.3\cdot 80^2 + 2\cdot 80)$$

$$3000 + 200 - 1920 - 160$$

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Panel 9

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6. Find the area under the graph $y = e^x + 2$ from x = -2 to x = 1. Sketch and shade the region. (6%)

$$\int_{-2}^{1} e^x + 2\,dx = e^x + 2x\Big|_{-2}^{1} = (e + 2) - (e^{-2} - 4) =$$

is positive

$$= e + 6 - e^{-2}$$

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Panel 10

9. Suppose the marginal cost of making q throw rugs is $c' = 8q - 3\sqrt{q} + 4e^q$, and the fixed cost is \$4400. Find the formula for the cost function. (6%)

$$C(x) = \int 8q - 3\sqrt{q}^{\,q^{1/2}} + 4e^q\,dq =$$

$$= 4q^2 - 3\cdot\tfrac{2}{3}q^{3/2} + 4e^q + C$$

$$= 4q^2 - 2q^{3/2} + 4e^q + C$$

$$C(0) = 0 - 0 + 4 + C = 4400$$

$$C = 4396$$

$$C(q) = 4q^2 - 2q^{3/2} + 4e^q + 4396$$

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Panel 11

Final Topics: Financial Mathematics

Recall Compound Interest Formula: If you invest a principal P at an interest rate r per period compounded for n periods in total, you have:

$$S = P(1+r)^n \quad , r \text{ is rate per period}$$

Ex: \$1000 at APR of 8% compounded quarterly for 5 years:

$$S = 1000\left(1+\frac{0.08}{4}\right)^{20} = \$1485.95$$

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Panel 12

Ex: Suppose \$500 is compounded semi-annually over 3 years and amounts to \$588.38. What is the nominal interest rate?

$$S = P(1+r)^n$$

$$588.38 = 500\left(1+\frac{r}{2}\right)^6$$

$$\frac{588.38}{500} = \left(1+\frac{r}{2}\right)^6$$

$$r = 5.5\%$$

$$1.17676 = \left(1+\frac{r}{2}\right)^6 \quad \Big|\sqrt[6]{\ }$$

$$\sqrt[6]{1.17676} = \left(1+\frac{r}{2}\right)$$

$$1.0275 = 1+\frac{r}{2} \Rightarrow 0.0275 = r/2 \Rightarrow 0.055 = r$$

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Panel 13

Ex: How long will it take for \$600 to amount to \$900 at APR of 6% compounded quarterly?

$S = P(1+r)^n$

$900 = 600\left(1+\frac{0.06}{4}\right)^{4t}$, t = # of years

$\frac{900}{600} = \left(1+\frac{0.06}{4}\right)^{4t}$

Quiz on open problems

$3/2 = \left(1+\frac{0.06}{4}\right)^{4t}$ | ln

$\ln(3/2) = \ln\left(\left(1+\frac{0.06}{4}\right)^{4t}\right) = 4t \ln(1.015)$

$\frac{\ln(1.5)}{\ln(1.015)} = 4t \Rightarrow t = \frac{1}{4}\frac{\ln(1.5)}{\ln(1.015)} = 6.8$ years $\rightarrow$ 7 years

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