Panel 1

Anti - Derivatives

$\int f(x)\,dx$ is that function $F(x)$ such that

integral of $f(x)\,dx$

$F'(x) = f(x)$

$\int$ = integral sign

$\int x^2\,dx = \frac{1}{3}x^3 + C$

Rules:

$$\int x^p\,dx = \frac{1}{p+1}x^{p+1} + C \quad , p \neq -1$$

$$\int \frac{1}{x}\,dx = \ln|x| + C \qquad \int e^x\,dx = e^x + C$$

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Panel 2

Ex: $\int 3x^3 + 4\sqrt{x} - \frac{7}{x^2} + \frac{3}{x} + 4e^x + 4\,dx$ $x^{1/2}$ x^{-2}

$3\frac{1}{4}x^4 + 4\frac{2}{3}x^{3/2} - 7(-1)x^{-1} + 3\ln|x| + 4e^x + 4x + C$

$\frac{3}{4}x^4 + \frac{8}{3}x^{3/2} + \frac{7}{x} + 3\ln|x| + 4e^x + 4x + C$

$\int x^2\,dx = \frac{1}{3}x^3 + C$

$\int \frac{1}{x^3}\,dx = \int x^{-3}\,dx = -\frac{1}{2}x^{-2} + C$

$\int \frac{1}{x}\,dx = \underline{\ln|x| + C}$

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Panel 3

$y = \int 3x^2 - \frac{7}{x} + 2e^x \, dx$ and $y(1) = 0$

$y = 3\frac{1}{3}x^3 - 7\ln|x| + 2e^x + C$

$y(1) = 1 - 7\cancel{\ln(1)}^{0} + 2e^1 + C = 0$

Remember: $\ln(1) = 0$

$1 + 2e + C = 0$

$C = -1 - 2e$

$y(x) = x^3 - 7\ln|x| + 2e^x - 1 - 2e$

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Panel 4

Marginal cost is $C'(q) = 3x^5 + 9\sqrt{x}$ $\quad 9x^{1/2} \quad 9 \cdot \frac{2}{3} x^{3/2}$

What is the cost? $C(q) = \int 3x^5 + 9\sqrt{x} \, dx$

$= 3\frac{1}{6}x^6 + 9\frac{2}{3}x^{3/2} + C$

$= \frac{1}{2}x^6 + 6x^{3/2} + C$

If fixed cost was \$100 : $C(0) = \frac{1}{2}0^6 + 6 \cdot 0^{3/2} + \boxed{C = 100}$

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Panel 5

Name: ______

Quiz #8

Evaluate the following indefinite integrals:

a) $\int 2x\,dx = x^2 + C$

b) $\int x^4\,dx = \frac{1}{5}x^5 + C$

c) $\int 9\sqrt[3]{x}\,dx = \int 9x^{1/3}\,dx = \underline{9\frac{3}{4}x^{4/3} + C}$

d) $\int \frac{3}{2x^2} - \frac{7}{3}\sqrt{x}\,dx = \int \frac{3}{2}x^{-2} - \frac{7}{3}x^{1/2}\,dx = \underline{\frac{3}{2}(-1)x^{-1} - \frac{7}{3}\frac{2}{3}x^{3/2} + C}$

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Panel 6

② Find a function $y(x)$ such that $y' = 4x - \frac{3}{x}$ and $y(1) = 2$

Hint: $\ln(1) = 0$

$y = \int 4x - \frac{3}{x}\,dx = 4\frac{1}{2}x^2 - 3\ln|x| + C$

$= 2x^2 - 3\ln(x) + C$

$y(1) = 2 - 0 + C = 2 \Rightarrow C = \underline{0}$

③ If a marginal cost function is $C'(q) = 10q - 3q^2$ and the fixed cost is \$10, find the total cost for producing $q = 3$ items.

$C(q) = \int 10q - 3q^2\,dq = 10\frac{1}{2}q^2 - 3\frac{1}{3}q^3 + C = 5q^2 - q^3 + C$

$C(0) = 0 - 0 + C = 10 \Rightarrow C = 10$

$C(3) = 5 \cdot 9 - 27 + 10 = \underline{28}$

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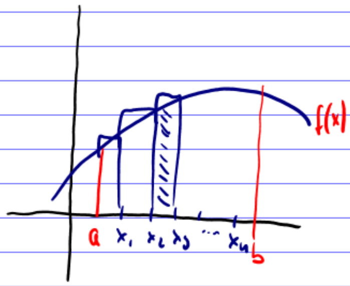
Panel 7

Definite Integral: $\int_a^b f(x)dx = \lim_{n\to\infty} \left(f(x_1)\Delta x_1 + f(x_2)\Delta x_2 + \dots + f(x_n)\Delta x_n \right)$

integral of f(x) dx from a to b



Remember: $\int_a^b f(x)dx$ is defined as a limit of little rectangle areas...

Fundamental Theorem of Calculus: If f is continuous

then $\int_a^b f(x)\,dx = F(b) - F(a)$, where F is anti derivative

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Panel 8

$$\int_a^b f(x)dx = \underbrace{F(x)}_{C}\Big|_a^b = F(b) - F(a)$$

C is not needed since it cancels out anyway.

Ex: $\int_0^2 2x\,dx = x^2 \cancel{+C}\Big|_0^2 = (2^2 \cancel{+C}) - (0^2 \cancel{+C}) = 4$

$$\int_0^1 5e^x - 7\sqrt[3]{x^5}\,dx = 5e^x - 7\cdot\frac{3}{8}x^{8/3}\Big|_0^1 = \left(5e - \frac{21}{8}\right) - (5 - 0)$$

$7x^{5/3}$

$= 5e - \frac{21}{8} - 5$

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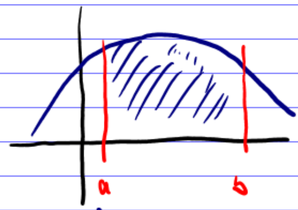
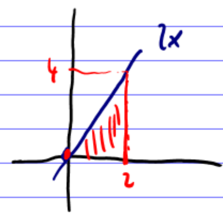
Panel 9

Interpretation of Definite Integral



$\int_a^b f(x)\,dx$ = area under curve from a to b as long as $f(x) \ge 0$

Ex: $\int_0^2 2x\,dx$ geometrically and algebraically.



$\int_0^2 2x\,dx = \frac{1}{2}$ base · height $= \frac{1}{2}\, 2 \cdot 4 = 4$

$\int_0^2 2x\,dx = x^2\Big|_0^2 = 2^2 - 0^2 = 4$

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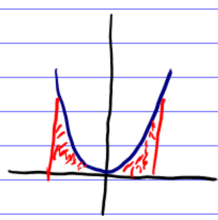
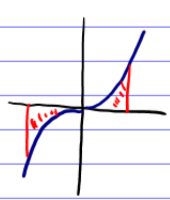
Panel 10

Ex: $$\int_{-1}^{1} x^2\,dx = \frac{1}{3}x^3\Big|_{-1}^{1} = \frac{1}{3}(1)^3 - \frac{1}{3}(-1)^3 = \frac{2}{3}$$



could use graph paper + count squares

Ex: $$\int_{-1}^{1} x^3\,dx = \frac{1}{4}x^4\Big|_{-1}^{1} = \frac{1}{4}(1)^4 - \frac{1}{4}(-1)^4 = 0$$



If f is not only positive, you get "signed" area

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Panel 11

$$\int f(x)\,dx = \text{antiderivative } F(x) + C \quad (\text{indefinite integral})$$

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a) \quad (\text{definite integral})$$

Ex: $$\int 3x^4 - \sqrt{x} + \frac{5}{x}\,dx = 3\,\frac{1}{5}x^5 - \frac{2}{3}x^{3/2} + 5\ln|x| + C$$

$$\int_0^1 x^3 - x^2\,dx = \frac{1}{4}x^4 - \frac{1}{3}x^3\Big|_0^1 = \left(\frac{1}{4} - \frac{1}{3}\right) - (0-0) = -\frac{1}{12}$$

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